1. SIGNED NUMBERS

The rules for operations with signed numbers are basic to successful work in algebra. Be sure you know, and can apply, the following rules.

Addition: To add numbers with the same sign, add the magnitudes of the numbers and keep the same sign. To add numbers with different signs, subtract the magnitudes of the numbers and use the sign of the number with the greater magnitude.

Example:

Add the following:

$$\begin{array}{r} +4 \\ \underline{+7} \\ +11 \end{array} \qquad \begin{array}{r} -4 \\ \underline{-7} \\ -11 \end{array} \qquad \begin{array}{r} -4 \\ \underline{+7} \\ +3 \end{array} \qquad \begin{array}{r} +4 \\ \underline{-7} \\ -3 \end{array}$$

Subtraction: Change the sign of the number to be subtracted and proceed with the rules for addition. Remember that subtracting is really adding the additive inverse.

Example:

Subtract the following:

$$\begin{array}{r} +4 \\ \underline{+7} \\ -3 \end{array} \qquad \begin{array}{r} -4 \\ \underline{-7} \\ +3 \end{array} \qquad \begin{array}{r} -4 \\ \underline{+7} \\ -11 \end{array} \qquad \begin{array}{r} +4 \\ \underline{-7} \\ +11 \end{array}$$

Multiplication: If there is an odd number of negative factors, the product is negative. An even number of negative factors gives a positive product.

Example:

Find the following products:

$(+4)(+7) = +28$ $\qquad$ $(-4)(-7) = +28$

$(+4)(-7) = -28$ $\qquad$ $(-4)(+7) = -28$

Division: If the signs are the same, the quotient is positive. If the signs are different, the quotient is negative.

Example:

Divide the following:

$$\frac{+28}{+4} = +7 \qquad \frac{-28}{+4} = -7$$

$$\frac{-28}{-4} = +7 \qquad \frac{+28}{-4} = -7$$

Exercise 1

Work out each problem. Circle the letter that appears before your answer.

1. At 8 a.m. the temperature was –4°. If the temperature rose 7 degrees during the next hour, what was the thermometer reading at 9 a.m.?
   (A) +11°
   (B) –11°
   (C) +7°
   (D) +3°
   (E) –3°

2. In Asia, the highest point is Mount Everest, with an altitude of 29,002 feet, while the lowest point is the Dead Sea, 1286 feet below sea level. What is the difference in their elevations?
   (A) 27,716 feet
   (B) 30,288 feet
   (C) 28,284 feet
   (D) 30,198 feet
   (E) 27,284 feet

3. Find the product of (–6)(–4)(–4) and (–2).
   (A) –16
   (B) +16
   (C) –192
   (D) +192
   (E) –98

4. The temperatures reported at hour intervals on a winter evening were +4°, 0°, –1°, –5°, and –8°. Find the average temperature for these hours.
   (A) –10°
   (B) –2°
   (C) +2°
   (D) $-2\frac{1}{2}°$
   (E) –3°

5. Evaluate the expression $5a - 4x - 3y$ if $a = -2$, $x = -10$, and $y = 5$.
   (A) +15
   (B) +25
   (C) –65
   (D) –35
   (E) +35

2. SOLUTION OF LINEAR EQUATIONS

Equations are the basic tools of algebra. The techniques of solving an equation are not difficult. Whether an equation involves numbers or only letters, the basic steps are the same.

1. If there are fractions or decimals, remove them by multiplication.
2. Remove any parentheses by using the distributive law.
3. Collect all terms containing the unknown for which you are solving on the same side of the equal sign. Remember that whenever a term crosses the equal sign from one side of the equation to the other, it must pay a toll. That is, it must change its sign.
4. Determine the coefficient of the unknown by combining similar terms or factoring when terms cannot be combined.
5. Divide both sides of the equation by the coefficient.

Example:

Solve for x: $5x - 3 = 3x + 5$

Solution:

$$2x = 8$$
$$x = 4$$

Example:

Solve for x: $\frac{2}{3}x - 10 = \frac{1}{4}x + 15$

Solution:

Multiply by 12. $8x - 120 = 3x + 180$

$$5x = 300$$
$$x = 60$$

Example:

Solve for x: $.3x + .15 = 1.65$

Solution:

Multiply by 100. $30x + 15 = 165$

$$30x = 150$$
$$x = 5$$

Example:

Solve for x: $ax - r = bx - s$

Solution:

$$ax - bx = r - s$$
$$x(a - b) = r - s$$
$$x = \frac{r - s}{a - b}$$

Example:

Solve for x: $6x - 2 = 8(x - 2)$

Solution:

$$6x - 2 = 8x - 16$$
$$14 = 2x$$
$$x = 7$$

Exercise 2

Work out each problem. Circle the letter that appears before your answer.

1. Solve for x: $3x - 2 = 3 + 2x$
   (A) 1
   (B) 5
   (C) –1
   (D) 6
   (E) –5

2. Solve for a: $8 - 4(a - 1) = 2 + 3(4 - a)$
   (A) $-\frac{5}{3}$
   (B) $-\frac{7}{3}$
   (C) 1
   (D) –2
   (E) 2

3. Solve for y: $\frac{1}{8}y + 6 = \frac{1}{4}y$
   (A) 48
   (B) 14
   (C) 6
   (D) 1
   (E) 2

4. Solve for x: $.02(x - 2) = 1$
   (A) 2.5
   (B) 52
   (C) 1.5
   (D) 51
   (E) 6

5. Solve for x: $4(x - r) = 2x + 10r$
   (A) $7r$
   (B) $3r$
   (C) r
   (D) $5.5r$
   (E) $2\frac{1}{3}r$

3. SIMULTANEOUS EQUATIONS IN TWO UNKNOWNS

In solving equations with two unknowns, it is necessary to work with two equations simultaneously. The object is to eliminate one of the unknowns, resulting in an equation with one unknown that can be solved by the methods of the previous section. This can be done by multiplying one or both equations by suitable constants in order to make the coefficients of one of the unknowns the same. Remember that multiplying *all* terms in an equation by the same constant does not change its value. The unknown can then be removed by adding or subtracting the two equations. When working with simultaneous equations, always be sure to have the terms containing the unknowns on one side of the equation and the remaining terms on the other side.

Example:

Solve for x: $7x + 5y = 15$
$5x - 9y = 17$

Solution:

Since we wish to solve for x, we would like to eliminate the y terms. This can be done by multiplying the top equation by 9 and the bottom equation by 5. In doing this, both y coefficients will have the same magnitude.

Multiplying the first by 9, we have

$63x + 45y = 135$

Multiplying the second by 5, we have

$25x - 45y = 85$

Since the y terms now have opposite signs, we can eliminate y by adding the two equations. If they had the same signs, we would eliminate by subtracting the two equations.

Adding, we have

$$\begin{aligned} 63x + 45y &= 135 \\ \underline{25x - 45y} &\underline{= 85} \\ 88x &= 220 \end{aligned}$$

$$x = \frac{220}{88} = 2\frac{1}{2}$$

Since we were only asked to solve for x, we stop here. If we were asked to solve for both x and y, we would now substitute $2\frac{1}{2}$ for x in either equation and solve the resulting equation for y.

$7(2.5) + 5y = 15$
$17.5 + 5y = 15$
$5y = -2.5$
$y = -.5$ or $-\frac{1}{2}$

Example:

Solve for x: $ax + by = r$
$cx - dy = s$

Solution:

Multiply the first equation by d and the second by b to eliminate the y terms by addition.

$$\begin{aligned} adx + bdy &= dr \\ \underline{bcx - bdy} &\underline{= bs} \\ adx + bcx &= dr + bs \end{aligned}$$

Factor out x to determine the coefficient of x.

$$x(ad + bc) = dr + bs$$

$$x = \frac{dr + bs}{ad + bc}$$

Exercise 3

Work out each problem. Circle the letter that appears before your answer.

1. Solve for x: $x - 3y = 3$
   $2x + 9y = 11$
   (A) 2
   (B) 3
   (C) 4
   (D) 5
   (E) 6

2. Solve for x: $.6x + .2y = 2.2$
   $.5x - .2y = 1.1$
   (A) 1
   (B) 3
   (C) 30
   (D) 10
   (E) 11

3. Solve for y: $2x + 3y = 12b$
   $3x - y = 7b$
   (A) $7\frac{1}{7}b$
   (B) $2b$
   (C) $3b$
   (D) $1\frac{2}{7}$
   (E) $-b$

4. If $2x = 3y$ and $5x + y = 34$, find y.
   (A) 4
   (B) 5
   (C) 6
   (D) 6.5
   (E) 10

5. If $x + y = -1$ and $x - y = 3$, find y.
   (A) 1
   (B) –2
   (C) –1
   (D) 2
   (E) 0

4. QUADRATIC EQUATIONS

In solving quadratic equations, there will always be two roots, even though these roots may be equal. A complete quadratic equation is of the form $ax^2 + bx + c = 0$, where a, b, and c are integers. At the level of this examination, $ax^2 + bx + c$ can always be factored. If b and/or c is equal to 0, we have an incomplete quadratic equation, which can still be solved by factoring and will still have two roots.

Example:

$x^2 + 5x = 0$

Solution:

Factor out a common factor of x.
$x(x + 5) = 0$
If the product of two factors is 0, either factor may be set equal to 0, giving $x = 0$ or $x + 5 = 0$.
From these two linear equations, we find the two roots of the given quadratic equation to be $x = 0$ and $x = –5$.

Example:

$6x^2 – 8x = 0$

Solution:

Factor out a common factor of $2x$.
$2x(3x – 4) = 0$
Set each factor equal to 0 and solve the resulting linear equations for x.

$2x = 0$	$3x – 4 = 0$
$x = 0$	$3x = 4$
	$x = \frac{4}{3}$

The roots of the given quadratic are 0 and $\frac{4}{3}$.

Example:

$x^2 – 9 = 0$

Solution:

$x^2 = 9$
$x = \pm 3$
Remember there must be two roots. This equation could also have been solved by factoring $x^2 – 9$ into $(x + 3)(x – 3)$ and setting each factor equal to 0. Remember that the difference of two perfect squares can always be factored, with one factor being the sum of the two square roots and the second being the difference of the two square roots.

Example:

$x^2 – 8 = 0$

Solution:

Since 8 is not a perfect square, this cannot be solved by factoring.
$x^2 = 8$
$x = \pm\sqrt{8}$
Simplifying the radical, we have $\sqrt{4} \cdot \sqrt{2}$, or $x = \pm 2\sqrt{2}$

Example:

$16x^2 - 25 = 0$

Solution:

Factoring, we have $(4x - 5)(4x + 5) = 0$

Setting each factor equal to 0, we have $x = \pm\frac{5}{4}$

If we had solved without factoring, we would have found $16x^2 = 25$

$$x^2 = \frac{25}{16}$$

$$x = \pm\frac{5}{4}$$

Example:

$x^2 + 6x + 8 = 0$

Solution:

$(x + 2)(x + 4) = 0$

If the last term of the trinomial is positive, both binomial factors must have the same sign, since the last two terms multiply to a positive product. If the middle term is also positive, both factors must be positive since they also add to a positive sum. Setting each factor equal to 0, we have $x = -4$ or $x = -2$

Example:

$x^2 - 2x - 15 = 0$

Solution:

We are now looking for two numbers that multiply to –15; therefore they must have opposite signs. To give –2 as a middle coefficient, the numbers must be –5 and +3.

$(x - 5)(x + 3) = 0$

This equation gives the roots 5 and –3.

Exercise 4

Work out each problem. Circle the letter that appears before your answer.

1. Solve for x: $x^2 - 8x - 20 = 0$
   (A) 5 and –4
   (B) 10 and –2
   (C) –5 and 4
   (D) –10 and –2
   (E) –10 and 2

2. Solve for x: $25x^2 - 4 = 0$
   (A) $\frac{4}{25}$ and $-\frac{4}{25}$
   (B) $\frac{2}{5}$ and $-\frac{2}{5}$
   (C) $\frac{2}{5}$ only
   (D) $-\frac{2}{5}$ only
   (E) none of these

3. Solve for x: $6x^2 - 42x = 0$
   (A) 7 only
   (B) –7 only
   (C) 0 only
   (D) 7 and 0
   (E) –7 and 0

4. Solve for x: $x^2 - 19x + 48 = 0$
   (A) 8 and 6
   (B) 24 and 2
   (C) –16 and –3
   (D) 12 and 4
   (E) none of these

5. Solve for x: $3x^2 = 81$
   (A) $9\sqrt{3}$
   (B) $\pm 9\sqrt{3}$
   (C) $3\sqrt{3}$
   (D) $\pm 3\sqrt{3}$
   (E) ± 9

5. EQUATIONS CONTAINING RADICALS

In solving equations containing radicals, it is important to get the radical alone on one side of the equation. Then square both sides to eliminate the radical sign. Solve the resulting equation. Remember that all solutions to radical equations must be checked, as squaring both sides may sometimes result in extraneous roots. In squaring each side of an equation, do not make the mistake of simply squaring each term. The entire side of the equation must be multiplied by itself.

Example:

$\sqrt{x-3} = 4$

Solution:

$x - 3 = 16$

$x = 19$

Checking, we have $\sqrt{16} = 4$, which is true.

Example:

$\sqrt{x-3} = -4$

Solution:

$x - 3 = 16$

$x = 19$

Checking, we have $\sqrt{16} = -4$, which is not true, since the radical sign means the principal, or positive, square root only. is 4, not –4; therefore, this equation has no solution.

Example:

$\sqrt{x^2-7} + 1 = x$

Solution:

First get the radical alone on one side, then square.

$\sqrt{x^2-7} = x - 1$

$x^2 - 7 = x^2 - 2x + 1$

$-7 = -2x + 1$

$2x = 8$

$x = 4$

Checking, we have $\sqrt{9} + 1 = 4$

$3 + 1 = 4$,

which is true.

Exercise 5

Work out each problem. Circle the letter that appears before your answer.

1. Solve for y: $\sqrt{2y} + 11 = 15$
   (A) 4
   (B) 2
   (C) 8
   (D) 1
   (E) no solution

2. Solve for x: $4\sqrt{2x-1} = 12$
   (A) 18.5
   (B) 4
   (C) 10
   (D) 5
   (E) no solution

3. Solve for x: $\sqrt{x^2-35} = 5-x$
   (A) 6
   (B) –6
   (C) 3
   (D) –3
   (E) no solution

4. Solve for y: $26 = 3\sqrt{2y} + 8$
   (A) 6
   (B) 18
   (C) 3
   (D) –6
   (E) no solution

5. Solve for x: $\sqrt{\frac{2x}{5}} = 4$
   (A) 10
   (B) 20
   (C) 30
   (D) 40
   (E) no solution