1. PART OF A WHOLE

A fraction represents a part of a whole. In dealing with fractional problems, we are usually dealing with a part of a quantity.

Example:

> Andrea and Danny ran for president of the Math Club. Andrea got 15 votes, while Danny got the other 10. What part of the votes did Andrea receive?

Solution:

> Andrea got 15 votes out of 25. That is $\frac{15}{25}$ or $\frac{3}{5}$ of the votes.

Exercise 1

Work out each problem. Circle the letter that appears before your answer.

1. In a class there are 18 boys and 12 girls. What part of the class is girls?

   (A) $\frac{2}{3}$

   (B) $\frac{3}{5}$

   (C) $\frac{2}{5}$

   (D) $\frac{1}{15}$

   (E) $\frac{3}{2}$

2. A team played 40 games and lost 6. What part of the games played did it win?

   (A) $\frac{3}{20}$

   (B) $\frac{3}{17}$

   (C) $\frac{14}{17}$

   (D) $\frac{17}{20}$

   (E) $\frac{7}{8}$

3. What part of an hour elapses between 3:45 p.m. and 4:09 p.m.?

   (A) $\frac{6}{25}$

   (B) $\frac{2}{5}$

   (C) $\frac{5}{12}$

   (D) $\frac{1}{24}$

   (E) 24

4. A camp employs 4 men, 6 women, 12 girls, and 8 boys. In the middle of the summer, 3 girls are fired and replaced by women. What part of the staff is then made up of women?

   (A) $\frac{1}{5}$

   (B) $\frac{2}{9}$

   (C) $\frac{1}{3}$

   (D) $\frac{3}{10}$

   (E) $\frac{1}{2}$

5. There are three times as many seniors as juniors at a high school Junior-Senior dance. What part of the students present are juniors?

   (A) $\dfrac{2}{5}$

   (B) $\dfrac{3}{5}$

   (C) $\dfrac{2}{3}$

   (D) $\dfrac{3}{4}$

   (E) $\dfrac{1}{4}$

6. What part of a yard is 1 ft. 3 in.?

   (A) $\dfrac{5}{12}$

   (B) $\dfrac{1}{3}$

   (C) $\dfrac{1}{2}$

   (D) $\dfrac{5}{8}$

   (E) $\dfrac{4}{9}$

7. Manorville High had a meeting of the Student Senate, which was attended by 10 freshmen, 8 sophomores, 15 juniors, and 7 seniors. What part of the students present at the meeting were sophomores?

   (A) $\dfrac{1}{4}$

   (B) $\dfrac{5}{8}$

   (C) $\dfrac{7}{40}$

   (D) $\dfrac{1}{5}$

   (E) $\dfrac{1}{3}$

8. The Dobkin family budgets its monthly income as follows: $\dfrac{1}{3}$ for food, $\dfrac{1}{4}$ for rent, $\dfrac{1}{10}$ for clothing, and $\dfrac{1}{5}$ for savings. What part is left for other expenses?

   (A) $\dfrac{3}{7}$

   (B) $\dfrac{1}{6}$

   (C) $\dfrac{7}{60}$

   (D) $\dfrac{2}{15}$

   (E) $\dfrac{3}{20}$

2. FINDING FRACTIONS OF FRACTIONS

Many problems require you to find a fractional part of a fractional part, such as $\frac{3}{5}$ of $\frac{2}{3}$. This involves multiplying the fractions together, $\frac{3}{4}$ of $\frac{2}{3}$ is $\frac{1}{2}$.

Example:

$\frac{1}{4}$ of the employees of Mr. Brown's firm earn over $20,000 per year. $\frac{1}{2}$ of the remainder earn between $15,000 and $20,000. What part of the employees earns less than $15,000 per year?

Solution:

$\frac{1}{4}$ earn over $20,000. $\frac{1}{2}$ of $\frac{3}{4}$ or $\frac{3}{8}$ earn between $15,000 and $20,000. That accounts for $\frac{1}{4} + \frac{3}{8}$ or $\frac{5}{8}$ of all employees. Therefore, the other $\frac{3}{8}$ earn less than $15,000.

Example:

A full bottle of isopropyl alcohol is left open in the school laboratory. If $\frac{1}{3}$ of the isopropyl alcohol evaporates in the first 12 hours and $\frac{2}{3}$ of the remainder evaporates in the second 12 hours, what part of the bottle is full at the end of 24 hours?

Solution:

$\frac{1}{3}$ evaporates during the first 12 hours. $\frac{2}{3}$ of $\frac{2}{3}$ or $\frac{4}{9}$ evaporates during the second 12 hours. This accounts for $\frac{7}{9}$ of the isopropyl alcohol. Therefore, $\frac{2}{9}$ of the bottle is still full.

Exercise 2

Work out each problem. Circle the letter that appears before your answer.

1. Mrs. Natt spent $\frac{2}{3}$ of the family income one year and divided the remainder between 4 different savings banks. If she put $2000 into each bank, what was the amount of her family income that year?

    (A) $8000
    (B) $16,000
    (C) $24,000
    (D) $32,000
    (E) $6000

2. After selling $\frac{2}{5}$ of the suits in his shop before Christmas, Mr. Gross sold the remainder of the suits at the same price per suit after Christmas for $4500. What was the income from the entire stock?

    (A) $3000
    (B) $7500
    (C) $1800
    (D) $2700
    (E) $8000

3. Of this year's graduating seniors at South High, $\frac{9}{10}$ will be going to college. Of these, $\frac{4}{5}$ will go to four-year colleges, while the rest will be going to two-year colleges. What part of the class will be going to two-year colleges?

   (A) $\frac{9}{50}$

   (B) $\frac{1}{5}$

   (C) $\frac{4}{5}$

   (D) $\frac{18}{25}$

   (E) $\frac{4}{25}$

4. Sue and Judy drove from New York to San Francisco, a distance of 3000 miles. They covered $\frac{1}{10}$ of the distance the first day and $\frac{2}{9}$ of the remaining distance the second day. How many miles were left to be driven?

   (A) 600
   (B) 2000
   (C) 2400
   (D) 2100
   (E) 2700

5. 800 employees work for the Metropolitan Transportation Company. $\frac{1}{4}$ of these are college graduates, while $\frac{5}{6}$ of the remainder are high school graduates. What part of the employees never graduated from high school?

   (A) $\frac{1}{6}$

   (B) $\frac{1}{8}$

   (C) $\frac{7}{8}$

   (D) $\frac{1}{12}$

   (E) $\frac{3}{4}$

3. FINDING WHOLE NUMBERS

When a fractional part of a number is given and we wish to find the number representing the whole, it is often easiest to translate the words into mathematical symbols and solve the resulting equation.

Example:

Norman buys a used car for $2400, which is $\frac{2}{5}$ of the original price. Find the original price.

Solution:

$$2400 \;=\; \frac{2}{5}x \quad \text{Multiply by 5.}$$
$$12000 \;=\; 2x$$
$$\$6000 \;=\; x$$

Example:

The gas gauge on Mary's car reads $\frac{1}{8}$ full. She asks the gasoline attendant to fill the tank and finds she needs 21 gallons. What is the capacity of her gas tank?

Solution:

$\frac{7}{8}$ of the tank is empty and requires 21 gallons to fill.

$$\frac{7}{8}x \;=\; 21 \quad \text{Multiply by 8.}$$
$$7x \;=\; 168$$
$$x \;=\; 24$$

Exercise 3

Work out each problem. Circle the letter that appears before your answer.

1. Daniel spent $4.50 for a ticket to the movies. This represents $\frac{3}{4}$ of his allowance for the week. What did he have left that week for other expenses?

   (A) $6.00
   (B) $4.00
   (C) $3.39
   (D) $1.13
   (E) $1.50

2. 350 seniors attended the prom. This represents $\frac{7}{9}$ of the class. How many seniors did not attend the prom?

   (A) 50
   (B) 100
   (C) 110
   (D) 120
   (E) 450

3. A resolution was passed by a ratio of 5:4. If 900 people voted for the resolution, how many voted against it?

   (A) 500
   (B) 400
   (C) 720
   (D) 600
   (E) 223

4. Mr. Rich owns $\frac{2}{7}$ of a piece of property. If the value of his share is $14,000, what is the total value of the property?

   (A) $70,000
   (B) $49,000
   (C) $98,000
   (D) $10,000
   (E) $35,000

5. The Stone family spends $500 per month for rent. This is $\frac{4}{15}$ of their total monthly income. Assuming that salaries remain constant, what is the Stone family income for one year?

   (A) $1875
   (B) $6000
   (C) $60,000
   (D) $22,500
   (E) $16,000

4. SOLVING WITH LETTERS

When problems use letters in place of numbers, the same principles discussed earlier apply. If you are not sure which operations to use, replace the letters with numbers to determine the steps needed in the solution.

Example:

It takes Mr. Cohen X days to paint his house. If he works for D days, what part of his house must still be painted?

Solution:

He has $X - D$ days of painting left to do out of a total of X days; therefore, $\dfrac{X - D}{X}$ is the correct answer.

Example:

Sue buys 500 stamps. X of these are 10-cent stamps. $\dfrac{1}{3}$ of the remainder are 15-cent stamps. How many 15-cent stamps does she buy?

Solution:

She buys $500 - X$ stamps that are not 10-cents stamps. $\dfrac{1}{3}$ of these are 15-cent stamps. Therefore, she buys $\dfrac{1}{3}(500 - X)$ or $\dfrac{500 - X}{3}$ 15-cent stamps.

Example:

John spent $\$X$ on the latest hit record album. This represents $\dfrac{1}{M}$ of his weekly allowance. What is his weekly allowance?

Solution:

Translate the sentence into an algebraic equation.

Let A = weekly allowance

$X = \dfrac{1}{M} \cdot A$ Multiply by M.

$MX = A$

Exercise 4

Work out each problem. Circle the letter that appears before your answer.

1. A class contains B boys and G girls. What part of the class is boys?

   (A) $\dfrac{B}{G}$

   (B) $\dfrac{G}{B}$

   (C) $\dfrac{B}{B+G}$

   (D) $\dfrac{B+G}{B}$

   (E) $\dfrac{B}{B-G}$

2. M men agreed to rent a ski lodge for a total of D dollars. By the time they signed the contract, the price had increased by $100. Find the amount each man had to contribute as his total share.

   (A) $\dfrac{D}{M}$

   (B) $\dfrac{D}{M}+100$

   (C) $\dfrac{D+100}{M}$

   (D) $\dfrac{M}{D}+100$

   (E) $\dfrac{M+100}{D}$

3. Of S students in Bryant High, $\dfrac{1}{3}$ study French. $\dfrac{1}{4}$ of the remainder study Italian. How many of the students study Italian?

   (A) $\dfrac{1}{6}S$

   (B) $\dfrac{1}{4}S$

   (C) $\dfrac{2}{3}S$

   (D) $\dfrac{1}{12}S$

   (E) $\dfrac{3}{7}S$

4. Mr. and Mrs. Feldman took t dollars in travelers checks with them on a trip. During the first week, they spent $\dfrac{1}{5}$ of their money. During the second week, they spent $\dfrac{1}{3}$ of the remainder. How much did they have left at the end of the second week?

   (A) $\dfrac{4t}{15}$

   (B) $\dfrac{t}{15}$

   (C) $\dfrac{7t}{15}$

   (D) $\dfrac{11t}{15}$

   (E) $\dfrac{8t}{15}$

5. Frank's gas tank was $\dfrac{1}{4}$ full. After putting in G gallons of gasoline, the tank was $\dfrac{7}{8}$ full. What was the capacity of the tank.

   (A) $\dfrac{5G}{8}$

   (B) $\dfrac{8G}{5}$

   (C) $\dfrac{8G}{7}$

   (D) $\dfrac{7G}{8}$

   (E) $4G$