Name:_______________________________

1. SEQUENCES INVOLVING EXPONENTIAL GROWTH (GEOMETRIC SEQUENCES)

In a sequence of terms involving exponential growth, which the testing service also calls a *geometric sequence*, there is a constant ratio between consecutive terms. In other words, each successive term is the same multiple of the preceding one. For example, in the sequence 2, 4, 8, 16, 32, . . . , notice that you multipy each term by 2 to obtain the next term, and so the constant ratio (multiple) is 2.

To solve problems involving geometric sequence, you can apply the following standard equation:

$$a \cdot r^{(n-1)} = T$$

In this equation:
The variable a is the value of the first term in the sequence
The variable r is the constant ratio (multiple)
The variable n is the position number of any particular term in the sequence
The variable T is the value of term n

If you know the values of any three of the four variables in this standard equation, then you can solve for the fourth one. (On the SAT, geometric sequence problems generally ask for the value of either a or T.)

Example (solving for T when a and r are given):

The first term of a geometric sequence is 2, and the constant multiple is 3. Find the second, third, and fourth terms.

Solution:

2nd term (T) = $2 \cdot 3^{(2-1)} = 2 \cdot 3^1 = 6$
3rd term (T) = $2 \cdot 3^{(3-1)} = 2 \cdot 3^2 = 2 \cdot 9 = 18$
4th term (T) = $2 \cdot 3^{(4-1)} = 2 \cdot 3^3 = 2 \cdot 27 = 54$

To solve for T when a and r are given, as an alternative to applying the standard equation, you can multiply a by $r^{(n-1)}$ times. Given $a = 2$ and $r = 3$:
2nd term (T) = $2 \cdot 3 = 6$
3rd term (T) = $2 \cdot 3 = 6 \cdot 3 = 18$
4th term (T) = $2 \cdot 3 = 6 \cdot 3 = 18 \cdot 3 = 54$

NOTE: Using the alternative method, you may wish to use your calculator to find T if a and/or r are large numbers.

Example (solving for a when r and T are given):

The fifth term of a geometric sequence is 768, and the constant multiple is 4. Find the 1st term (a).

Solution:

$$a \times 4^{(5-1)} = 768$$
$$a \times 4^4 = 768$$
$$a \times 256 = 768$$
$$a = \frac{768}{256}$$
$$a = 3$$

Example (solving for T when a and another term in the sequence are given):

To find a particular term (T) in a geometric sequence when the first term and another term are given, first determine the constant ratio (r), and then solve for T. For example, assume that the first and sixth terms of a geometric sequence are 2 and 2048, respectively. To find the value of the fourth term, first apply the standard equation to determine r :

Solution:

$$2 \times r^{(6-1)} = 2048$$
$$2 \times r^5 = 2048$$
$$r^5 = \frac{2048}{2}$$
$$r^5 = 1024$$
$$r = \sqrt[5]{1024}$$
$$r = 4$$

The constant ratio is 4. Next, in the standard equation, let $a = 2$, $r = 4$, and $n = 4$, and then solve for T :

$$2 \times 4^{(4-1)} = T$$
$$2 \times 4^3 = T$$
$$2 \times 64 = T$$
$$128 = T$$

The fourth term in the sequence is 128.

Exercise 1

Work out each problem. For questions 1–3, circle the letter that appears before your answer. Questions 4 and 5 are grid-in questions.

1. On January 1, 1950, a farmer bought a certain parcel of land for $1,500. Since then, the land has doubled in value every 12 years. At this rate, what will the value of the land be on January 1, 2010?

   (A) $7,500
   (B) $9,000
   (C) $16,000
   (D) $24,000
   (E) $48,000

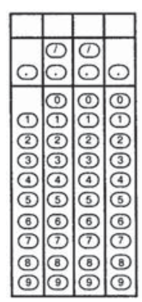
2. A certain type of cancer cell divides into two cells every four seconds. How many cells are observable 32 seconds after observing a total of four cells?

   (A) 1,024
   (B) 2,048
   (C) 4,096
   (D) 5,512
   (E) 8,192

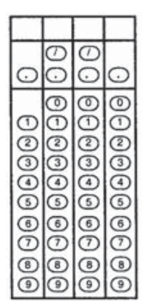
3. The seventh term of a geometric sequence with constant ratio 2 is 448. What is the first term of the sequence?

   (A) 6
   (B) 7
   (C) 8
   (D) 9
   (E) 11

4. Three years after an art collector purchases a certain painting, the value of the painting is $2,700. If the painting increased in value by an average of 50 percent per year over the three year period, how much did the collector pay for the painting, in dollars?



5. What is the second term in a geometric series with first term 3 and third term 147?



2. SETS (UNION, INTERSECTION, ELEMENTS)

A *set* is simply a collection of elements; elements in a set are also referred to as the “members” of the set. An SAT problem involving sets might ask you to recognize either the union or the intersection of two (or more) sets of numbers.

The *union* of two sets is the set of all members of either or both sets. For example, the union of the set of all negative integers and the set of all non-negative integers is the set of all integers. The *intersection* of two sets is the set of all common members – in other words, members of *both* sets. For example, the intersection of the set of integers less than 11 and the set of integers greater than 4 but less than 15 is the following set of six consecutive integers: {5,6,7,8,9,10}.

On the new SAT, a problem involving either the union or intersection of sets might apply any of the following concepts: the real number line, integers, multiples, factors (including prime factors), divisibility, or counting.

Example:

Set A is the set of all positive multiples of 3, and set B is the set of all positive multiples of 6. What is the union and intersection of the two sets?

Solution:

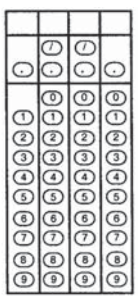
The union of sets A and B is the set of all postitive multiples of 3.
The intersection of sets A and B is the set of all postitive multiples of 6.

Exercise 2

Work out each problem. Note that question 2 is a grid-in question. For all other questions, circle the letter that appears before your answer.

1. Which of the following describes the union of the set of integers less than 20 and the set of integers greater than 10?
   (A) Integers 10 through 20
   (B) All integers greater than 10 but less than 20
   (C) All integers less than 10 and all integers greater than 20
   (D) No integers
   (E) All integers

2. Set A consists of the positive factors of 24, and set B consists of the positive factors of 18. The intersection of sets A and B is a set containing how many members?



3. The union of sets X and Y is a set that contains exactly two members. Which of the following pairs of sets could be sets X and Y ?
   (A) The prime factors of 15; the prime factors of 30
   (B) The prime factors of 14; the prime factors of 51
   (C) The prime factors of 19; the prime factors of 38
   (D) The prime factors of 22; the prime factors of 25
   (E) The prime factors of 39; the prime factors of 52

4. The set of all multiples of 10 could be the intersection of which of the following pairs of sets?
   (A) The set of all multiples of $\frac{5}{2}$; the set of all multiples of 2
   (B) The set of all multiples of $\frac{3}{5}$; the set of all multiples of 5
   (C) The set of all multiples of $\frac{3}{2}$; the set of all multiples of 10
   (D) The set of all multiples of $\frac{3}{4}$; the set of all multiples of 2
   (E) The set of all multiples of $\frac{5}{2}$; the set of all multiples of 4

5. For all real numbers x, sets P, Q, and R are defined as follows:

   P:$\{x \geq -10\}$

   Q:$\{x \geq 10\}$

   R:$\{|x| \leq 10\}$

   Which of the following indicates the intersection of sets P, Q, and R ?
   (A) x = any real number
   (B) $x \geq -10$
   (C) $x \geq 10$
   (D) $x = 10$
   (E) $-10 \leq x \leq 10$

3. ABSOLUTE VALUE

The *absolute value* of a real number refers to the number's distance from zero (the origin) on the real-number line. The absolute value of x is indicated as $|x|$. The absolute value of a negative number always has a positive value.

Example:

$|-2 - 3| - |2 - 3| =$
(A) –2
(B) –1
(C) 0
(D) 1
(E) 4

Solution:

The correct answer is (E). $|-2 - 3| = |-5| = 5$, and $|2 - 3| = |-1| = 1$. Performing subtraction: $5 - 1 = 4$.

The concept of absolute value can be incorporated into many different types of problems on the new SAT, including those involving algebraic expressions, equations, and inequalities, as well as problems involving functional notation and the graphs of functions.

Exercise 3

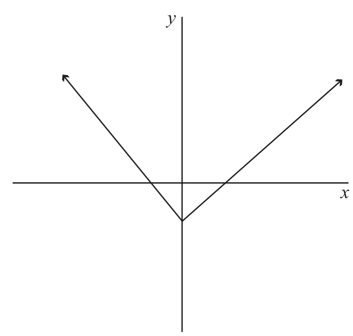
Work out each problem. Circle the letter that appears before your answer.

1. $|7 - 2| - |2 - 7| =$
   (A) –14
   (B) –9
   (C) –5
   (D) 0
   (E) 10

2. For all integers a and b, where $b \neq 0$, subtracting b from a must result in a positive integer if:
   (A) $|a - b|$ is a positive integer
   (B) $\left(\frac{a}{b}\right)$ is a positive integer
   (C) $(b - a)$ is a negative integer
   (D) $(a + b)$ is a positive integer
   (E) (ab) is a positive integer

3. What is the complete solution set for the inequality $|x - 3| > 4$?
   (A) $x > -1$
   (B) $x > 7$
   (C) $-1 < x < 7$
   (D) $x < -7, x > 7$
   (E) $x < -1, x > 7$

4. The figure below shows the graph of a certain equation in the xy-plane.



   Which of the following could be the equation?
   (A) $x = |y| - 1$
   (B) $y = |x| - 1$
   (C) $|y| = x - 1$
   (D) $y = x + 1$
   (E) $|x| = y - 1$

5. If $f(x) = \left|\frac{1}{x} - 3\right| - x$, then $f\left(\frac{1}{2}\right) =$
   (A) –1
   (B) $-\frac{1}{2}$
   (C) 0
   (D) $\frac{1}{2}$
   (E) 1

4. EXPONENTS (POWERS)

An *exponent*, or *power*, refers to the number of times that a number (referred to as the *base* number) is multiplied by itself, plus 1. In the number 2^3, the base number is 2 and the exponent is 3. To calculate the value of 2^3, you multiply 2 by itself twice: $2^3 = 2 \cdot 2 \cdot 2 = 8$. In the number $\left(\frac{2}{3}\right)^4$, the base number is $\frac{2}{3}$ and the exponent is 4. To calculate the value of $\left(\frac{2}{3}\right)^4$, you multiply $\frac{2}{3}$ by itself three times: $\left(\frac{2}{3}\right)^4 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{16}{81}$.

An SAT problem might require you to combine two or more terms that contain exponents. Whether you can you combine base numbers—using addition, subtraction, multiplication, or division—*before* applying exponents to the numbers depends on which operation you're performing. When you add or subtract terms, you cannot combine base numbers or exponents:

$$a^x + b^x \neq (a + b)^x$$
$$a^x - b^x \neq (a - b)^x$$

Example:

If $x = -2$, then $x^5 - x^2 - x =$
(A) 26
(B) 4
(C) –34
(D) –58
(E) –70

Solution:

The correct answer is (C). You cannot combine exponents here, even though the base number is the same in all three terms. Instead, you need to apply each exponent, in turn, to the base number, then subtract:

$$x^5 - x^2 - x = (-2)^5 - (-2)^2 - (-2) = -32 - 4 + 2 = -34$$

There are two rules you need to know for combining exponents by multiplication or division. First, you can combine base numbers first, but only if the exponents are the same:

$$a^x \cdot b^x = (ab)^x$$

$$\frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x$$

Second, you can combine exponents first, but only if the base numbers are the same. When multiplying these terms, add the exponents. When dividing them, subtract the denominator exponent from the numerator exponent:

$$a^x \cdot a^y = a^{(x + y)}$$

$$\frac{a^x}{a^y} = a^{(x-y)}$$

When the same base number (or term) appears in both the numerator and denominator of a fraction, you can factor out, or cancel, the number of powers common to both.

Example:

Which of the following is a simplified version of $\frac{x^2y^3}{x^3y^2}$?

(A) $\frac{y}{x}$

(B) $\frac{x}{y}$

(C) $\frac{1}{xy}$

(D) 1

(E) x^5y^5

Solution:

The correct answer is (A). The simplest approach to this problem is to cancel, or factor out, x^2 and y^2 from numerator and denominator. This leaves you with x^1 in the denominator and y^1 in the denominator.

You should also know how to raise exponential numbers to powers, as well as how to raise base numbers to negative and fractional exponents. To raise an exponential number to a power, multiply exponents together:

$$\left(a^x\right)^y = a^{xy}$$

Raising a base number to a negative exponent is equivalent to 1 divided by the base number raised to the exponent's absolute value:

$$a^{-x} = \frac{1}{a^x}$$

To raise a base number to a fractional exponent, follow this formula:

$$a^{\frac{x}{y}} = \sqrt[y]{a^x}$$

Also keep in mind that any number other than 0 (zero) raised to the power of 0 (zero) equals 1:

$$a^0 = 1 \ [a \neq 0]$$

Example:

$(2^3)^2 \cdot 4^{-3} =$

(A) 16

(B) 1

(C) $\frac{2}{3}$

(D) $\frac{1}{2}$

(E) $\frac{1}{8}$

Solution:

The correct answer is (B). $(2^3)^2 \cdot 4^{-3} = 2^{(2)(3)} \cdot \frac{1}{4^3} = \frac{2^6}{4^3} = \frac{2^6}{2^6} = 1$

Exercise 4

Work out each problem. For questions 1–4, circle the letter that appears before your answer. Question 5 is a grid-in question.

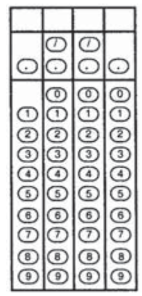
1. $\frac{a^2b}{b^2c} \div \frac{a^2c}{bc^2} =$
   (A) $\frac{1}{a}$
   (B) $\frac{1}{b}$
   (C) $\frac{b}{a}$
   (D) $\frac{c}{b}$
   (E) 1

2. $4^n + 4^n + 4^n + 4^n =$
   (A) 4^{4n}
   (B) 16^n
   (C) $4^{(n \cdot n \cdot n \cdot n)}$
   (D) $4^{(n+1)}$
   (E) 16^{4n}

3. Which of the following expressions is a simplified form of $(-2x^2)^4$?
   (A) $16x^8$
   (B) $8x^6$
   (C) $-8x^8$
   (D) $-16x^6$
   (E) $-16x^8$

4. If $x = -1$, then $x^{-3} + x^{-2} + x^2 + x^3 =$
   (A) –2
   (B) –1
   (C) 0
   (D) 1
   (E) 2

5. What integer is equal to $4^{3/2} + 4^{3/2}$?



5. FUNCTION NOTATION

In a *function* (or *functional relationship*), the value of one variable depends upon the value of, or is "a function of," another variable. In mathematics, the relationship can be expressed in various forms. The new SAT uses the form $y = f(x)$—where y is a function of x. (Specific variables used may differ.) To find the value of the function for any value x, substitute the x-value for x wherever it appears in the function.

Example:

If $f(x) = 2x - 6x$, then what is the value of $f(7)$?

Solution:

The correct answer is –28. First, you can combine $2x - 6x$, which equals $-4x$. Then substitute (7) for x in the function: $-4(7) = -28$. Thus, $f(7) = -28$.

A problem on the new SAT may ask you to find the value of a function for either a number value (such as 7, in which case the correct answer will also be a number value) or for a variable expression (such as $7x$, in which case the correct answer will also contain the variable x). A more complex function problem might require you to apply two different functions or to apply the same function twice, as in the next example.

Example:

If $f(x) = \frac{2}{x^2}$, then $f\left(\frac{1}{2}\right) \times f\left(\frac{1}{x}\right) =$

(A) $4x$

(B) $\frac{1}{8x}$

(C) $16x$

(D) $\frac{1}{4x^2}$

(E) $16x^2$

Solution:

The correct answer is (E). Apply the function to each of the two x-values (in the first instance, you'll obtain a numerical value, while in the second instance you'll obtain an variable expression:

$$f\left(\frac{1}{2}\right) = \frac{2}{\left(\frac{1}{2}\right)^2} = \frac{2}{\frac{1}{4}} = 2 \times 4 = 8$$

$$f\left(\frac{1}{x}\right) = \frac{2}{\left(\frac{1}{x}\right)^2} = \frac{2}{\left(\frac{1}{x^2}\right)} = 2x^2$$

Then, combine the two results according to the operation specified in the question:

$$f\left(\frac{1}{2}\right) \times f\left(\frac{1}{x}\right) = 8 \times 2x^2 = 16x^2$$

Exercise 5

Work out each problem. Circle the letter that appears before your answer.

1. If $f(x) = 2x\sqrt{x}$, then for which of the following values of x does $f(x) = x$?

   (A) $\frac{1}{4}$
   (B) $\frac{1}{2}$
   (C) 2
   (D) 4
   (E) 8

2. If $f(a) = a^{-3} - a^{-2}$, then $f\left(\frac{1}{3}\right) =$

   (A) $-\frac{1}{6}$
   (B) $\frac{1}{6}$
   (C) 6
   (D) 9
   (E) 18

3. If $f(x) = x^2 + 3x - 4$, then $f(2 + a) =$

   (A) $a^2 + 7a + 6$
   (B) $2a^2 - 7a - 12$
   (C) $a^2 + 12a + 3$
   (D) $6a^2 + 3a + 7$
   (E) $a^2 - a + 6$

4. If $f(x) = x^2$ and $g(x) = x + 3$, then $g(f(x)) =$

   (A) $x + 3$
   (B) $x^2 + 6$
   (C) $x + 9$
   (D) $x^2 + 3$
   (E) $x^3 + 3x^2$

5. If $f(x) = \frac{x}{2}$, then $f(x^2) \div \left(f(x)\right)^2 =$

   (A) x^3
   (B) 1
   (C) $2x^2$
   (D) 2
   (E) $2x$

6. FUNCTIONS—DOMAIN AND RANGE

A function consists of a *rule* along with two sets—called the *domain* and the *range*. The domain of a function $f(x)$ is the set of all values of x on which the function $f(x)$ is defined, while the range of $f(x)$ is the set of all values that result by applying the rule to all values in the domain.

By definition, a function must assign *exactly one* member of the range to each member of the domain, and must assign at least one member of the domain to each member of the range. Depending on the function's rule and its domain, the domain and range might each consist of a finite number of values; or either the domain or range (or both) might consist of an infinite number of values.

Example:

In the function $f(x) = x + 1$, if the domain of x is the set {2,4,6}, then applying the rule that $f(x) = x + 1$ to all values in the domain yields the function's range: the set {3,5,7}. (All values other than 2, 4, and 6 are outside the domain of x, while all values other than 3, 5, and 7 are outside the function's range.)

Example:

In the function $f(x) = x^2$, if the domain of x is the set of all real numbers, then applying the rule that $f(x) = x^2$ to all values in the domain yields the function's range: the set of all non-negative real numbers. (Any negative number would be outside the function's range.)

Exercise 6

Work out each problem. Circle the letter that appears before your answer.

1. If $f(x) = \sqrt{x+1}$, and if the domain of x is the set {3,8,15}, then which of the following sets indicates the range of $f(x)$?

   (A) $\{-4, -3, -2, 2, 3, 4\}$
   (B) $\{2, 3, 4\}$
   (C) $\{4, 9, 16\}$
   (D) $\{3, 8, 15\}$
   (E) {all real numbers}

2. If $f(a) = 6a - 4$, and if the domain of a consists of all real numbers defined by the inequality $-6 < a < 4$, then the range of $f(a)$ contains all of the following members EXCEPT:

   (A) -24
   (B) $\sqrt{\frac{1}{6}}$
   (C) 0
   (D) 4
   (E) 20

3. If the range of the function $f(x) = x^2 - 2x - 3$ is the set $R = \{0\}$, then which of the following sets indicates the largest possible domain of x ?

   (A) $\{-3\}$
   (B) $\{3\}$
   (C) $\{-1\}$
   (D) $\{3, -1\}$
   (E) all real numbers

4. If $f(x) = \sqrt{x^2 - 5x + 6}$, which of the following indicates the set of all values of x at which the function is NOT defined?

   (A) $\{x \mid x < 3\}$
   (B) $\{x \mid 2 < x < 3\}$
   (C) $\{x \mid x < -2\}$
   (D) $\{x \mid -3 < x < 2\}$
   (E) $\{x \mid x < -3\}$

5. If $f(x) = \sqrt[3]{\frac{1}{x}}$, then the largest possible domain of x is the set that includes

   (A) all non-zero integers.
   (B) all non-negative real numbers.
   (C) all real numbers except 0.
   (D) all positive real numbers.
   (E) all real numbers.

7. LINEAR FUNCTIONS—EQUATIONS AND GRAPHS

A *linear function* is a function f given by the general form $f(x) = mx + b$, in which m and b are constants. In algebraic functions, especially where defining a line on the xy-plane is involved, the variable y is often used to represent $f(x)$, and so the general form becomes $y = mx + b$. In this form, each x-value (member of the domain set) can be paired with its corresponding y-value (member of the range set) by application of the function.

Example:

In the function $y = 3x + 2$, if the domain of x is the set of all positive integers less than 5, then applying the function over the entire domain of x results in the following set of (x,y) pairs: $S = \{(1,5), (2,8), (3,11), (4,14)\}$.

In addition to questions requiring you to solve a system of linear equations (by using either the substitution or addition-subtraction method), the new SAT includes questions requiring you to recognize any of the following:

- A linear function (equation) that defines two or more particular (x,y) pairs (members of the domain set and corresponding members of the range set). These questions sometimes involve real-life situations; you may be asked to construct a mathematical "model" that defines a relationship between, for example, the price of a product and the number of units of that product.
- The graph of a particular linear function on the xy-plane
- A linear function that defines a particular line on the xy-plane.

Variations on the latter two types of problems may involve determining the slope and/or y-intercept of a line defined by a function, or identifying a function that defines a given slope and y-intercept.

Example:

In the linear function f, if $f(-3) = 3$ and if the slope of the graph of f in the xy-plane is 3, what is the equation of the graph of f ?

(A) $y = 3x - 3$
(B) $y = 3x + 12$
(C) $y = x - 6$
(D) $y = -x$
(E) $y = 3x - 12$

Solution:

The correct answer is (B). In the general equation $y = mx + b$, slope (m) is given as 3. To determine b, substitute -3 for x and 3 for y, then solve for b: $3 = 3(-3) + b$; $12 = b$. Only in choice (B) does $m = 3$ and $b = 12$.

Exercise 7

Work out each problem. Circle the letter that appears before your answer.

1. XYZ Company pays its executives a starting salary of $80,000 per year. After every two years of employment, an XYZ executive receives a salary raise of $1,000. Which of the following equations best defines an XYZ executive's salary (S) as a function of the number of years of employment (N) at XYZ?

   (A) $S = \frac{1{,}000}{N} + 80{,}000$

   (B) $S = N + 80{,}000$

   (C) $S = \frac{80{,}000}{N} + 1{,}000$

   (D) $S = 1{,}000N + 80{,}000$

   (E) $S = 500N + 80{,}000$

2. In the linear function g, if $g(4) = -9$ and $g(-2) = 6$, what is the y-intercept of the graph of g in the xy-plane?

   (A) $-\frac{9}{2}$

   (B) $-\frac{5}{2}$

   (C) $\frac{2}{5}$

   (D) 1

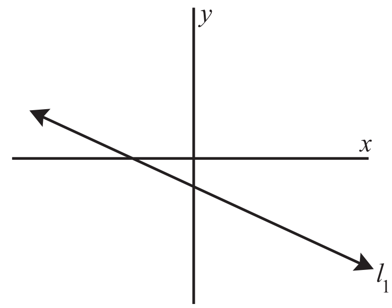
   (E) $\frac{3}{2}$

3. If two linear function f and g have identical domains and ranges, which of the following, each considered individually, could describe the graphs of f and g in the xy-plane?

   I. two parallel lines
   II. two perpendicular lines
   III. two vertical lines

   (A) I only
   (B) I and II only
   (C) II only
   (D) II and III only
   (E) I, II, and III

4. In the xy-plane below, if the scales on both axes are the same, which of the following could be the equation of a function whose graph is l_1 ?



   (A) $y = \frac{2}{3}x - 3$
   (B) $y = -2x + 1$
   (C) $y = x + 3$
   (D) $y = -3x - \frac{2}{3}$
   (E) $y = -\frac{2}{3}x - 3$

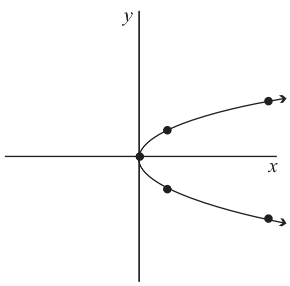
5. If h is a linear function, and if $h(2) = 3$ and $h(4) = 1$, then $h(-101) =$

   (A) –72
   (B) –58
   (C) 49
   (D) 92
   (E) 106

8. QUADRATIC FUNCTIONS—EQUATIONS AND GRAPHS

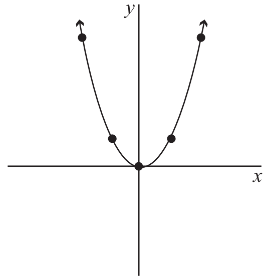
In Chapter 8, you learned to solve quadratic equations in the general form $ax^2 + bx + c = 0$ by factoring the expression on the left-hand side of this equation to find the equation's two roots— the values of x that satisfy the equation. (Remember that the two roots might be the same.) The new SAT may also include questions involving *quadratic functions* in the general form $f(x) = ax^2 + bx + c$. (Note that a, b, and c are constants and that a is the only essential constant.) In quadratic functions, especially where defining a graph on the xy-plane is involved, the variable y is often used to represent $f(x)$, and x is often used to represent $f(y)$.

The graph of a quadratic equation of the basic form $y = ax^2$ or $x = ay^2$ is a *parabola*, which is a U-shaped curve. The point at which the dependent variable is at its minimum (or maximum) value is the *vertex*. In each of the following four graphs, the parabola's vertex lies at the origin (0,0). Notice that the graphs are constructed by tabulating and plotting several (x,y) pairs, and then connecting the points with a smooth curve:



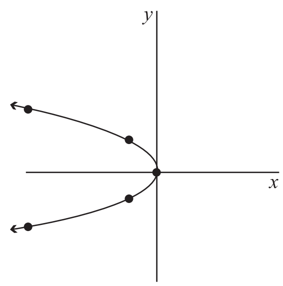
$x = y^2$

x	y
4	–2
1	–1
0	0
1	1
4	2



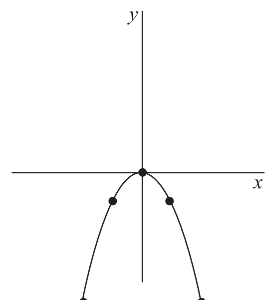
$y = x^2$

x	y
–2	4
–1	1
0	0
1	1
2	4



$x = -y^2$

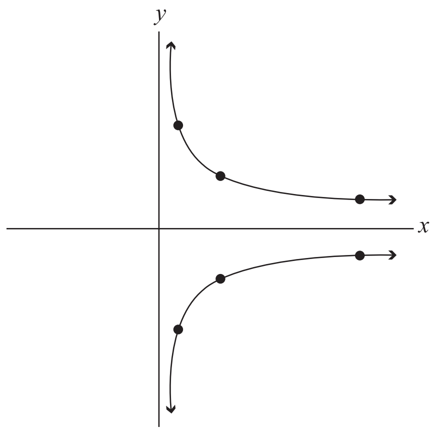
x	y
–4	–2
–1	–1
0	0
–1	1
–4	2

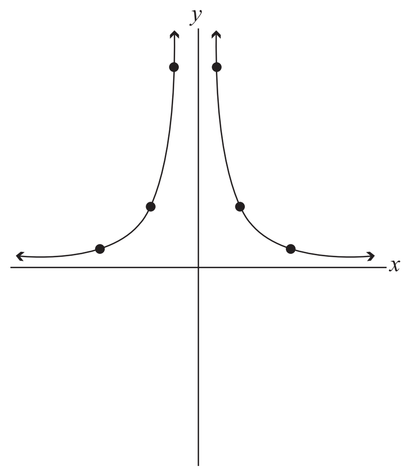


$y = -x^2$

x	y
–2	–4
–1	–1
0	0
1	–1
2	–4

The graph of a quadratic equation of the basic form $x = \frac{1}{y^2}$ or $y = \frac{1}{x^2}$ is a *hyperbola*, which consists of two U-shaped curves that are symmetrical about a particular line, called the *axis of symmetry*. The axis of symmetry of the graph of $x = \frac{1}{y^2}$ is the x-axis, while the axis of symmetry in the graph of $y = \frac{1}{x^2}$ is the y-axis, as the next figure shows. Again, the graphs are constructed by tabulating and plotting some (x,y) pairs, then connecting the points:





$x = \frac{1}{y^2}$

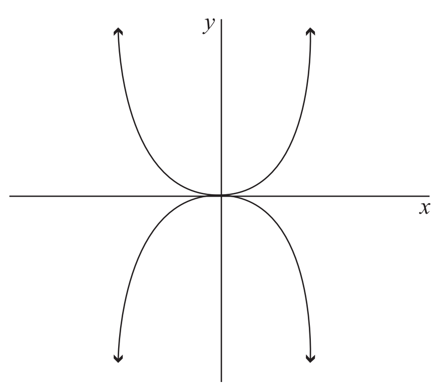
x	y
$\frac{1}{4}$	–2
1	–1
4	$-\frac{1}{2}$
4	$\frac{1}{2}$
1	1
$\frac{1}{4}$	2

$y = \frac{1}{x^2}$

x	y
–2	$\frac{1}{4}$
–1	1
$-\frac{1}{2}$	4
$\frac{1}{2}$	4
1	1
2	$\frac{1}{4}$

The new SAT might include a variety of question types involving quadratic functions—for example, questions that ask you to recognize a quadratic equation that defines a particular graph in the *xy*-plane or to identify certain features of the graph of a quadratic equation, or compare two graphs

Example:



The graph shown in the *xy*-plane above could represent which of the following equations?

(A) $|x^2| = |y^2|$

(B) $x = |y^2|$

(C) $|y| = x^2$

(D) $y = |x^2|$

(E) $|x| = y^2$

Solution:

The correct answer is (C). The equation $|y| = x^2$ represents the union of the two equations $y = x^2$ and $-y = x^2$. The graph of $y = x^2$ is the parabola extending upward from the origin (0,0) in the figure, while the graph of $-y = x^2$ is the parabola extending downward from the origin.

Example:

In the *xy*-plane, the graph of $y + 2 = \frac{x^2}{2}$ shows a parabola that opens

(A) downward.
(B) upward.
(C) to the right.
(D) to the left.
(E) either upward or downward.

Solution:

The correct answer is (B). Plotting three or more points of the graph on the *xy*-plane should show the parabola's orientation. First, it is helpful to isolate *y* in the equation $y = \frac{x^2}{2} - 2$. In this equation, substitute some simple values for *x* and solve for *y* in each case. For example, substituting 0, 2, and –2 for *x* gives us the three (*x*,*y*) pairs (0,–2), (2,0), and (–2,0). Plotting these three points on the *xy*-plane, then connecting them with a curved line, suffices to show a parabola that opens upward.

An SAT question might also ask you to identify a quadratic equation that defines two or more domain members and the corresponding members of the function's range (these questions sometimes involve "models" of real-life situations).

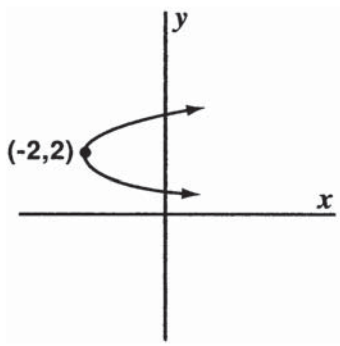
Exercise 8

Work out each problem. Circle the letter that appears before your answer.

1. Which of the following equations defines a function containing the (x,y) pairs (1,–1), (2,–4), (3,–9), and (4,–16) ?

(A) $y = -2x$
(B) $y = 2x$
(C) $y = x^2$
(D) $y = -x^2$
(E) $y = -2x^2$

2. The figure below shows a parabola in the xy-plane.



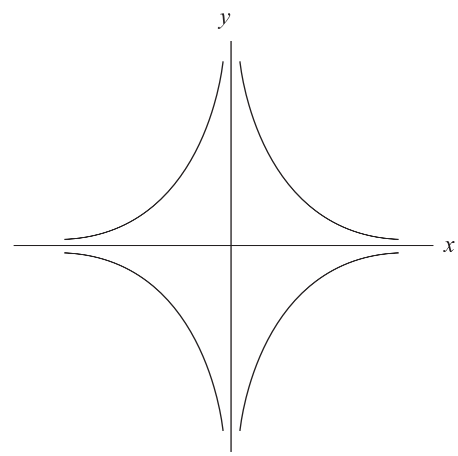
Which of the following equations does the graph best represent?

(A) $x = (y - 2)^2 - 2$
(B) $x = (y + 2)^2 - 2$
(C) $x = -(y - 2)^2 - 2$
(D) $y = (x - 2)^2 + 2$
(E) $y = (x - 2)^2 - 2$

3. In the xy-plane, which of the following is an equation whose graph is the graph of $y = \frac{x^2}{3}$ translated three units horizontally and to the left?

(A) $y = x^2$
(B) $y = \frac{x^2}{3} + 3$
(C) $y = \frac{x^2}{3} - 3$
(D) $y = \frac{(x-3)^2}{3}$
(E) $y = \frac{(x+3)^2}{3}$

4. Which of the following is the equations best defines the graph shown below in the xy-plane?



(A) $y = \frac{1}{x^2}$
(B) $x = \left|\frac{1}{y^2}\right|$
(C) $x = \frac{1}{y^2}$
(D) $|x| = \frac{1}{y^2}$
(E) $y = \left|\frac{1}{x^2}\right|$

5. ABC Company projects that it will sell 48,000 units of product X per year at a unit price of \$1, 12,000 units per year at \$2 per unit, and 3,000 units per year at \$4 per unit. Which of the following equations could define the projected number of units sold per year (N), as a function of price per unit (P)?

(A) $N = \frac{48{,}000}{P^2 + 2}$
(B) $N = \frac{48{,}000}{P^2}$
(C) $N = \frac{48{,}000}{P + 14}$
(D) $N = \frac{48{,}000}{P + 4}$
(E) $N = \frac{48{,}000}{P^2 + 8}$