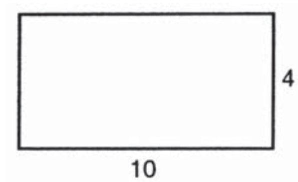
Name: ____________________

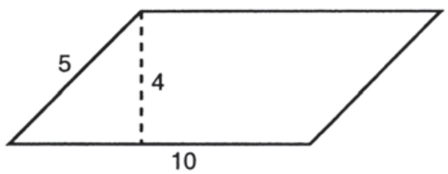
1. AREAS

A. Rectangle = base · altitude = *bh*



Area = 40

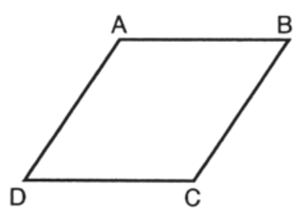
B. Parallelogram = base · altitude = *bh*



Area = 40

Notice that the altitude is different from the side. It is always shorter than the second side of the parallelogram, as a perpendicular is the shortest distance from a point to a line.

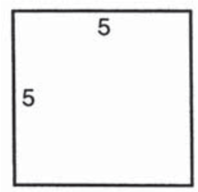
C. Rhombus = $\frac{1}{2}$ · product of the diagonals = $\frac{1}{2}d_1d_2$



If $AC = 20$ and $BD = 30$, the area of $ABCD = \frac{1}{2}(20)(30) = 300$

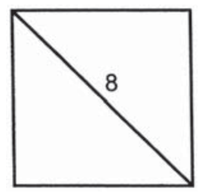
D. Square = side · side = s^2

Area = 25



Remember that every square is a rhombus, so that the rhombus formula may be used for a square if the diagonal is given. The diagonals of a square are equal.

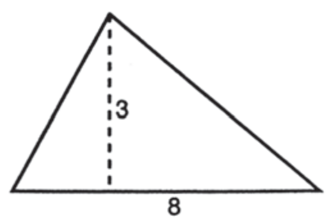
Area = $\frac{1}{2}(8)(8) = 32$



Remember also that a rhombus is *not* a square. Therefore do not use the s^2 formula for a rhombus. A rhombus, however, is a parallelogram, so you may use *bh* if you do not know the diagonals.

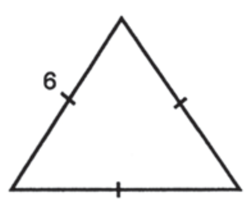
E. Triangle = $\frac{1}{2}$ · base · altitude = $\frac{1}{2}bh$

$A = \frac{1}{2}(8)(3) = 12$



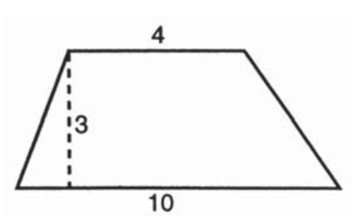
F. Equilateral Triangle = $\frac{1}{4}$ · side squared · $\sqrt{3} = \frac{s^2}{4}\sqrt{3}$

$A = \frac{36}{4}\sqrt{3} = 9\sqrt{3}$



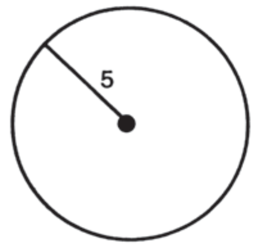
G. Trapezoid = $\frac{1}{2}$ · altitude · sum of bases = $\frac{1}{2}h(b_1 + b_2)$

$A = \frac{1}{2}(3)(14) = 21$



H. Circle = $\pi \cdot$ radius squared = $\pi \cdot r^2$

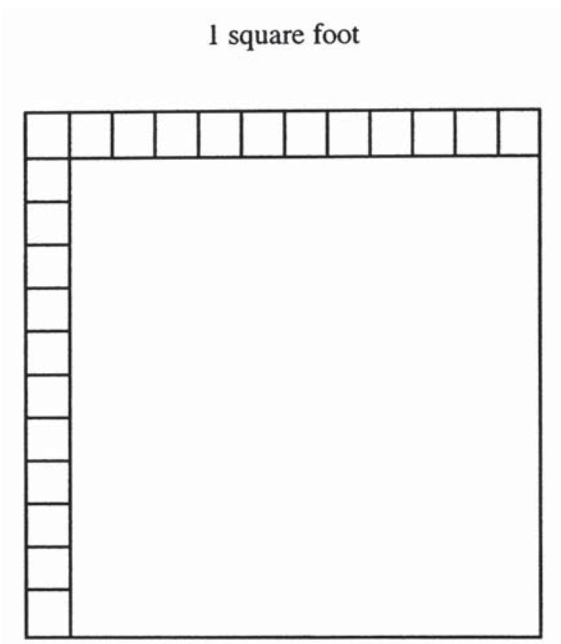
$A = \pi \cdot (5)^2 = 25\pi$



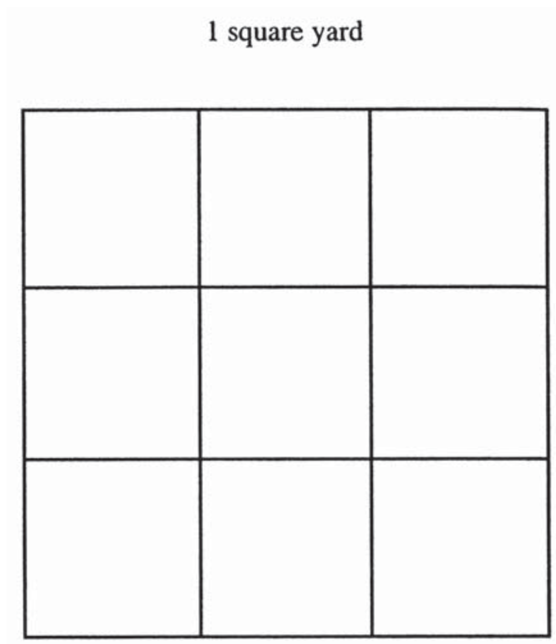
Remember that π is the ratio of the circumference of any circle and its diameter. $\pi = \frac{c}{d}$. The approximations you have used for π in the past (3.14 or $\frac{22}{7}$) are just that—approximations. π is an irrational number and cannot be expressed as a fraction or terminating decimal. Therefore all answers involving π should be left in terms of π unless you are given a specific value to substitute for π.

A word about units—Area is measured in square units. That is, we wish to compute how many squares one inch on each side (a square inch) or one foot on each side (a square foot), etc., can be used to cover a given surface. To change from square inches to square feet or square yards, remember that

144 square inches = 1 square foot
9 square feet = 1 square yard



12" = 1'
12 one inch squares in a row
12 rows
144 square inches in 1 sq. ft.



3' = 1 yd.
3 one foot squares in a row
3 rows
9 square feet in 1 sq. yd.

Exercise 1

Work out each problem. Circle the letter that appears before your answer.

1. The dimensions of a rectangular living room are 18 feet by 20 feet. How many square yards of carpeting are needed to cover the floor?
   (A) 360
   (B) 42
   (C) 40
   (D) 240
   (E) 90

2. In a parallelogram whose area is 15, the base is represented by $x + 7$ and the altitude is $x - 7$. Find the base of the parallelogram.
   (A) 8
   (B) 15
   (C) 1
   (D) 34
   (E) 5

3. The sides of a right triangle are 6, 8, and 10. Find the altitude drawn to the hypotenuse.
   (A) 2.4
   (B) 4.8
   (C) 3.4
   (D) 3.5
   (E) 4.2

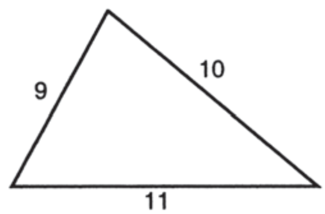
4. If the diagonals of a rhombus are represented by $4x$ and $6x$, the area may be represented by
   (A) $6x$
   (B) $24x$
   (C) $12x$
   (D) $6x^2$
   (E) $12x^2$

5. A circle is inscribed in a square whose side is 6. Express the area of the circle in terms of π.
   (A) 6π
   (B) 3π
   (C) 9π
   (D) 36π
   (E) 12π

2. PERIMETER

The perimeter of a figure is the distance around the outside. If you were fencing in an area, the number of feet of fencing you would need is the perimeter. Perimeter is measured in linear units, that is, centimeters, inches, feet, meters, yards, etc.

A. Any polygon = sum of all sides



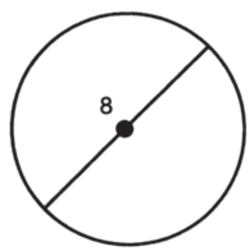
$P = 9 + 10 + 11 = 30$

B. Circle = π · diameter = π*d*

or

$2 \cdot \pi \cdot \text{radius} = 2\pi r$

Since $2r = d$, these formulas are the same. The perimeter of a circle is called its circumference.

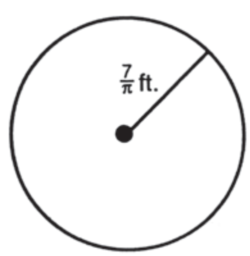


$C = \pi \cdot 8 = 8\pi$

or

$C = 2 \cdot \pi \cdot 4 = 8\pi$

The distance covered by a wheel in one revolution is equal to the circumference of the wheel. In making one revolution, every point on the rim comes in contact with the ground. The distance covered is then the same as stretching the rim out into a straight line.



The distance covered by this wheel in one revolution is $2 \cdot \pi \cdot \frac{7}{\pi} = 14$ feet.

Exercise 2

Work out each problem. Circle the letter that appear before your answer.

1. The area of an equilateral triangle is $16\sqrt{3}$. Find its perimeter.
   (A) 24
   (B) 16
   (C) 48
   (D) $24\sqrt{3}$
   (E) $48\sqrt{3}$

2. The hour hand of a clock is 3 feet long. How many feet does the tip of this hand move between 9:30 P.M. and 1:30 A.M. the following day?
   (A) π
   (B) 2π
   (C) 3π
   (D) 4π
   (E) 24π

3. If the radius of a circle is increased by 3, the circumference is increased by
   (A) 3
   (B) 3π
   (C) 6
   (D) 6π
   (E) 4.5

4. The radius of a wheel is 18 inches. Find the number of feet covered by this wheel in 20 revolutions.
   (A) 360π
   (B) 360
   (C) 720π
   (D) 720
   (E) 60π

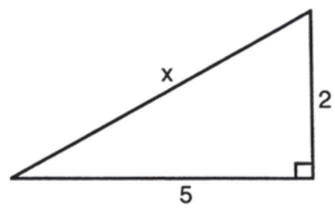
5. A square is equal in area to a rectangle whose base is 9 and whose altitude is 4. Find the perimeter of the square.
   (A) 36
   (B) 26
   (C) 13
   (D) 24
   (E) none of these

3. RIGHT TRIANGLES

A. Pythagorean theorem

$(\text{leg})^2 + (\text{leg})^2 = (\text{hypotenuse})^2$

$$(5)^2 + (2)^2 = x^2$$
$$25 + 4 = x^2$$
$$29 = x^2$$
$$x = \sqrt{29}$$



B. Pythagorean triples

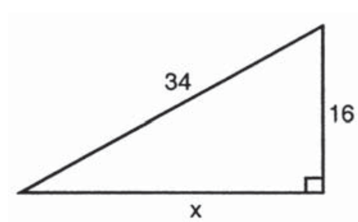
These are sets of numbers that satisfy the Pythagorean Theorem. When a given set of numbers such as 3, 4, 5 forms a Pythagorean triple ($3^2 + 4^2 = 5^2$), any multiples of this set such as 6, 8, 10 or 30, 40, 50 also form a Pythagorean triple. Memorizing the sets of Pythagorean triples that follow will save you valuable time in solving problems, for, if you recognize given numbers as multiples of Pythagorean triples, you do not have to do any arithmetic at all. The most common Pythagorean triples that should be memorized are

3, 4, 5

5, 12, 13

8, 15, 17

7, 24, 25



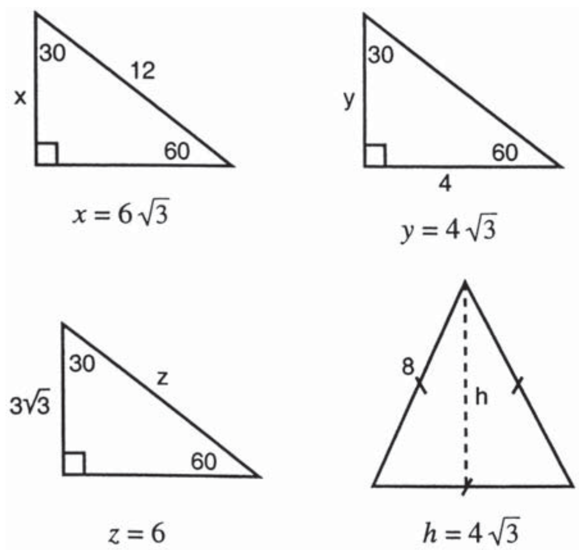
Squaring 34 and 16 to apply the Pythagorean theorem would take too much time. Instead, recognize the hypotenuse as 2(17). Suspect an 8, 15, 17 triangle. Since the given leg is 2(8), the missing leg will be 2(15) or 30, without any computation at all.

C. 30°–60°–90° triangle

a) The leg opposite the 30° angle is one-half the hypotenuse.

b) The leg opposite the 60° angle is one-half the hypotenuse $\cdot \sqrt{3}$.

c) An altitude in an equilateral triangle forms a 30°–60°–90° triangle and is therefore equal to one-half the side $\cdot \sqrt{3}$.

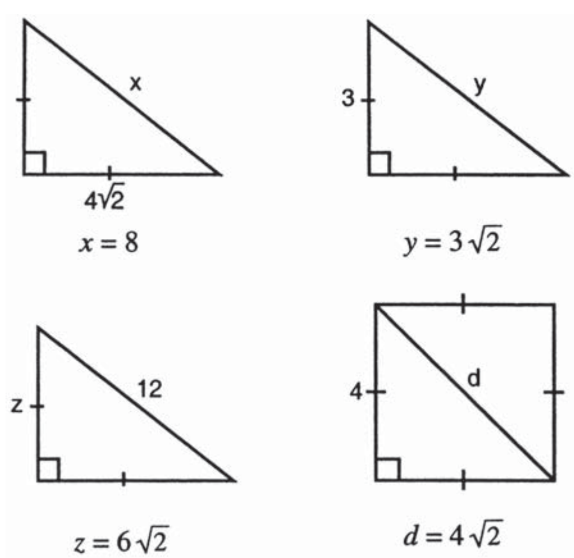


D. 45°–45°–90° triangle (isosceles right triangle)

a) Each leg is one-half the hypotenuse times $\sqrt{2}$.

b) Hypotenuse is leg times $\sqrt{2}$.

c) The diagonal of a square forms a 45°–45°–90° triangle and is therefore equal to a side times $\sqrt{2}$.



Exercise 3

Work out each problem. Circle the letter that appears before your answer.

1. A farmer uses 140 feet of fencing to enclose a rectangular field. If the ratio of length to width is 3 : 4, find the diagonal, in feet, of the field.

   (A) 50
   (B) 100
   (C) 20
   (D) 10
   (E) cannot be determined

2. Find the altitude of an equilateral triangle whose side is 20.

   (A) 10
   (B) $20\sqrt{3}$
   (C) $10\sqrt{3}$
   (D) $20\sqrt{2}$
   (E) $10\sqrt{2}$

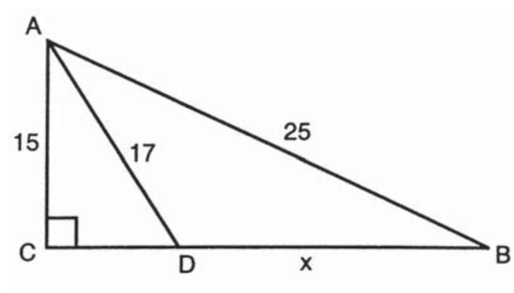
3. Two boats leave the same dock at the same time, one traveling due west at 8 miles per hour and the other due north at 15 miles per hour. How many miles apart are the boats after three hours?

   (A) 17
   (B) 69
   (C) 75
   (D) 51
   (E) 39

4. Find the perimeter of a square whose diagonal is $6\sqrt{2}$.

   (A) 24
   (B) $12\sqrt{2}$
   (C) 12
   (D) 20
   (E) $24\sqrt{2}$

5. Find the length of *DB*.



   (A) 8
   (B) 10
   (C) 12
   (D) 15
   (E) 20

4. COORDINATE GEOMETRY

A. Distance between two points =

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The distance between (–3, 2) and (5, –1) is

$$\sqrt{[-3-5]^2 + [2-(-1)]^2} = \sqrt{(-8)^2 + (3)^2} = \sqrt{64+9} = \sqrt{73}$$

B. The midpoint of a line segment =

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Since a midpoint is in the middle, its coordinates are found by averaging the *x* coordinates and averaging the *y* coordinates. Remember that to find the average of two numbers, you add them and divide by two. Be very careful of signs in adding signed numbers. Review the rules given earlier if necessary.

The midpoint of the segment joining (–4, 1) to (–2, –9) is

$$\left(\frac{-4+(-2)}{2}, \frac{1+(-9)}{2}\right) = \left(\frac{-6}{2}, \frac{-8}{2}\right) = (-3,-4)$$

Exercise 4

Work out each problem. Circle the letter that appears before your answer.

1. *AB* is the diameter of a circle whose center is *O*. If the coordinates of *A* are (2, 6) and the coordinates of *B* are (6, 2), find the coordinates of *O*.
   - (A) (4, 4)
   - (B) (4, –4)
   - (C) (2, –2)
   - (D) (0, 0)
   - (E) (2, 2)

2. *AB* is the diameter of a circle whose center is *O*. If the coordinates of *O* are (2, 1) and the coordinates of *B* are (4, 6), find the coordinates of *A*.
   - (A) $\left(3, 3\frac{1}{2}\right)$
   - (B) $\left(1, 2\frac{1}{2}\right)$
   - (C) (0, –4)
   - (D) $\left(2\frac{1}{2}, 1\right)$
   - (E) $\left(-1, -2\frac{1}{2}\right)$

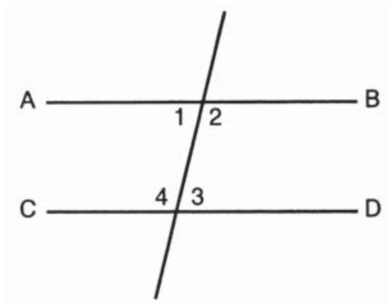
3. Find the distance from the point whose coordinates are (4, 3) to the point whose coordinates are (8, 6).
   - (A) 5
   - (B) 25
   - (C) $\sqrt{7}$
   - (D) $\sqrt{67}$
   - (E) 15

4. The vertices of a triangle are (2, 1), (2, 5), and (5, 1). The area of the triangle is
   - (A) 12
   - (B) 10
   - (C) 8
   - (D) 6
   - (E) 5

5. The area of a circle whose center is at (0,0) is 16π. The circle passes through each of the following points *except*
   - (A) (4, 4)
   - (B) (0, 4)
   - (C) (4, 0)
   - (D) (–4, 0)
   - (E) (0, –4)

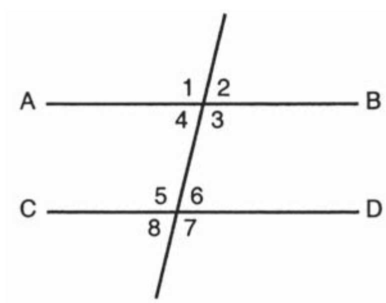
5. PARALLEL LINES

A. If two lines are parallel and cut by a transversal, the alternate interior angles are congruent.



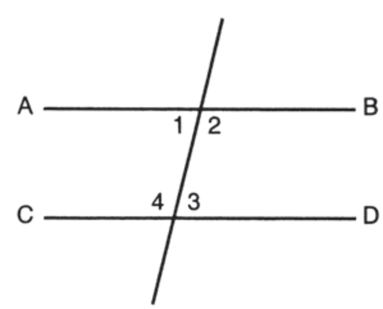
If $\overline{AB}$ is parallel to $\overline{CD}$, then angle 1 ≅ angle 3 and
angle 2 ≅ angle 4.

B. If two parallel lines are cut by a transversal, the corresponding angles are congruent.



If $\overline{AB}$ is parallel to $\overline{CD}$, then angle 1 ≅ angle 5
angle 2 ≅ angle 6
angle 3 ≅ angle 7
angle 4 ≅ angle 8

C. If two parallel lines are cut by a transversal, interior angles on the same side of the transversal are supplementary.

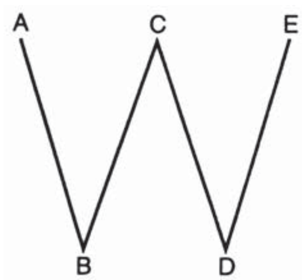


If $\overline{AB}$ is parallel to $\overline{CD}$, angle 1 + angle 4 = 180°
angle 2 + angle 3 = 180°

Exercise 5

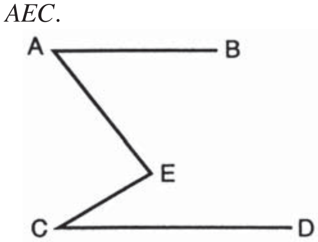
Work out each problem. Circle the letter that appears before your answer.

1. If $\overline{AB}$ is parallel to $\overline{CD}$, $\overline{BC}$ is parallel to $\overline{ED}$, and angle $B = 30°$, find the number of degrees in angle *D*.



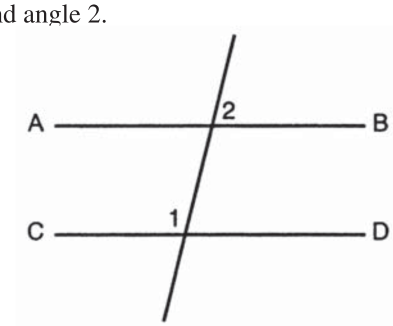
(A) 30
(B) 60
(C) 150
(D) 120
(E) none of these

2. If $\overline{AB}$ is parallel to $\overline{CD}$, angle $A = 35°$, and angle $C = 45°$, find the number of degrees in angle *AEC*.



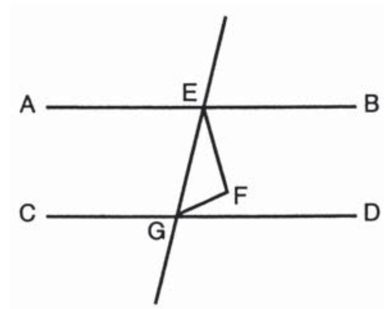
(A) 35
(B) 45
(C) 70
(D) 80
(E) 100

3. If $\overline{AB}$ is parallel to $\overline{CD}$ and angle $1 = 130°$, find angle 2.



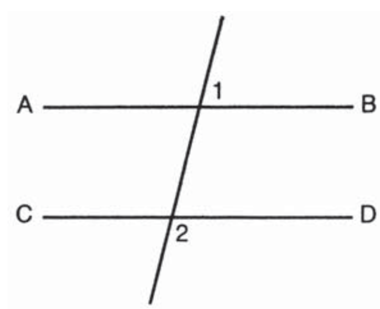
(A) 130°
(B) 100°
(C) 40°
(D) 60°
(E) 50°

4. If $\overline{AB}$ is parallel to $\overline{CD}$, $\overline{EF}$ bisects angle *BEG*, and $\overline{GF}$ bisects angle *EGD*, find the number of degrees in angle *EFG*.



(A) 40
(B) 60
(C) 90
(D) 120
(E) cannot be determined

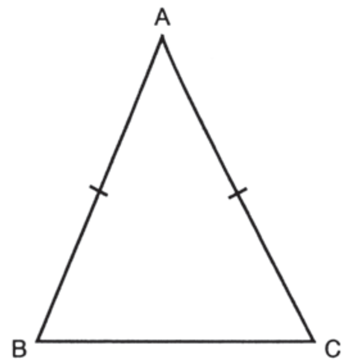
5. If $\overline{AB}$ is parallel to $\overline{CD}$ and angle $1 = x°$, then the sum of angle 1 and angle 2 is



(A) $2x°$
(B) $(180 - x)°$
(C) 180°
(D) $(180 + x)°$
(E) none of these

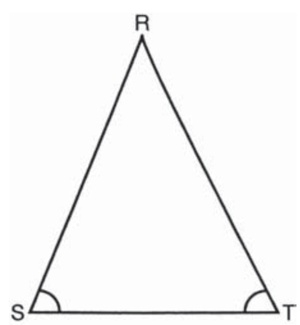
6. TRIANGLES

A. If two sides of a triangle are congruent, the angles opposite these sides are congruent.



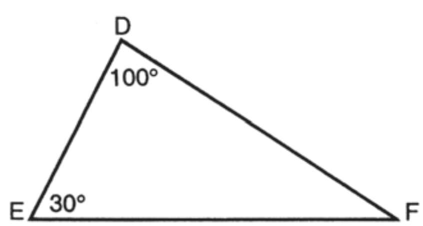
If $\overline{AB} \cong \overline{AC}$, then angle $B \cong$ angle C.

B. If two angles of a triangle are congruent, the sides opposite these angles are congruent.



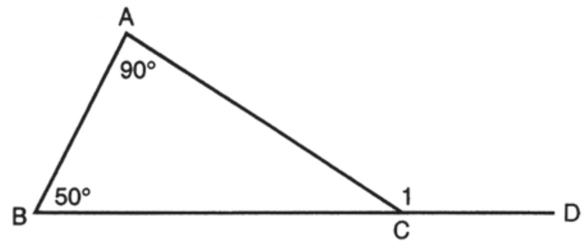
If angle $S \cong$ angle T, then $\overline{RS} \cong \overline{RT}$.

C. The sum of the measures of the angles of a triangle is 180°.



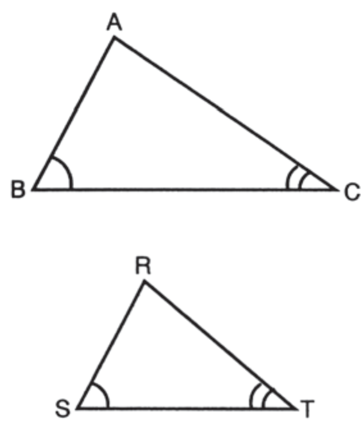
Angle $F = 180° - 100° - 30° = 50°$.

D. The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.



Angle 1 = 140°

E. If two angles of one triangle are congruent to two angles of a second triangle, the third angles are congruent.



Angle *A* will be congruent to angle *R*.

Exercise 6

Work out each problem. Circle the letter that appears before your answer.

1. The angles of a triangle are in the ratio 1 : 5 : 6. This triangle is
   (A) acute
   (B) obtuse
   (C) isosceles
   (D) right
   (E) equilateral

2. If the vertex angle of an isosceles triangle is 50°, find the number of degrees in one of the base angles.
   (A) 50
   (B) 130
   (C) 60
   (D) 65
   (E) 55

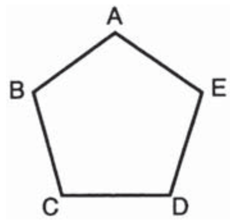
3. In triangle *ABC*, angle *A* is three times as large as angle *B*. The exterior angle at *C* is 100°. Find the number of degrees in angle *A*.
   (A) 60
   (B) 80
   (C) 20
   (D) 25
   (E) 75

4. If a base angle of an isosceles triangle is represented by $x°$, represent the number of degrees in the vertex angle.
   (A) $180 – x$
   (B) $x – 180$
   (C) $2x – 180$
   (D) $180 – 2x$
   (E) $90 – 2x$

5. In triangle *ABC*, $\overline{AB} = \overline{BC}$. If angle $A = (4x – 30)°$ and angle $C = (2x + 10)°$, find the number of degrees in angle *B*.
   (A) 20
   (B) 40
   (C) 50
   (D) 100
   (E) 80

7. POLYGONS

A. The sum of the measures of the angles of a polygon of *n* sides is $(n - 2)180°$.



Since *ABCDE* has 5 sides, angle *A* + angle *B* + angle *C* + angle *D* + angle *E* = $(5 - 2)180° = 3(180)° = 540°$

B. Properties of a parallelogram

a) Opposite sides are parallel

b) Opposite sides are congruent

c) Opposite angles are congruent

d) Consecutive angles are supplementary

e) Diagonals bisect each other

C. Properties of a rectangle

a) All 5 properties of a parallelogram

b) All angles are right angles

c) Diagonals are congruent

D. Properties of a rhombus

a) All 5 properties of a parallelogram

b) All sides are congruent

c) Diagonals are perpendicular to each other

d) Diagonals bisect the angles

E. Properties of a square

a) All 5 parallelogram properties

b) Two additional rectangle properties

c) Three additional rhombus properties

Exercise 7

Work out each problem. Circle the letter that appears before your answer.

1. Find the number of degrees in the sum of the interior angles of a hexagon.

   (A) 360
   (B) 540
   (C) 720
   (D) 900
   (E) 1080

2. In parallelogram $ABCD$, $AB = x + 4$, $BC = x - 6$, and $CD = 2x - 16$. Find AD.

   (A) 20
   (B) 24
   (C) 28
   (D) 14
   (E) 10

3. In parallelogram $ABCD$, $AB = x + 8$, $BC = 3x$, and $CD = 4x - 4$. $ABCD$ must be a

   (A) rectangle
   (B) rhombus
   (C) trapezoid
   (D) square
   (E) pentagon

4. The sum of the angles in a rhombus is

   (A) 180°
   (B) 360°
   (C) 540°
   (D) 720°
   (E) 450°

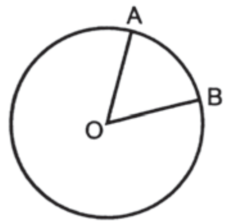
5. Which of the following statements is *false?*

   (A) A square is a rhombus.
   (B) A rhombus is a parallelogram.
   (C) A rectangle is a rhombus.
   (D) A rectangle is a parallelogram.
   (E) A square is a rectangle.

8. CIRCLES

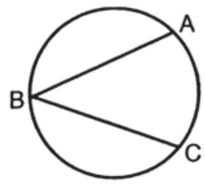
A. A central angle is equal in degrees to its intercepted arc.

If arc $AB = 50°$, then angle $AOB = 50°$.



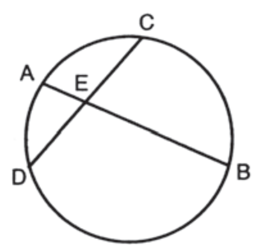
B. An inscribed angle is equal in degrees to one-half its intercepted arc.

If arc $AC = 100°$, then angle $ABC = 50°$.



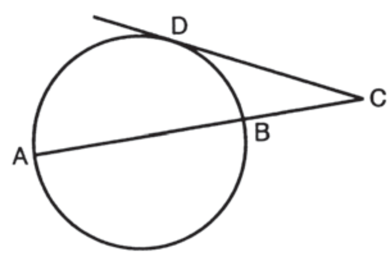
C. An angle formed by two chords intersecting in a circle is equal in degrees to one-half the sum of its intercepted arcs.

If arc $AD = 30°$ and arc $CB = 120°$, then angle $AED = 75°$.



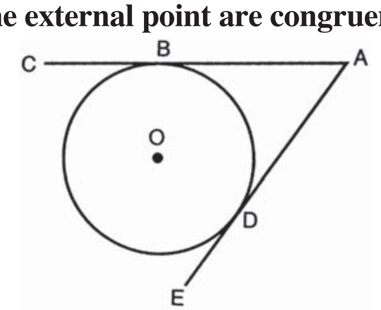
D. An angle outside the circle formed by two secants, a secant and a tangent, or two tangents is equal in degrees to one-half the difference of its intercepted arcs.

If arc $AD = 120°$ and arc $BD = 30°$, then angle $C = 45°$.



E. Two tangent segments drawn to a circle from the same external point are congruent.

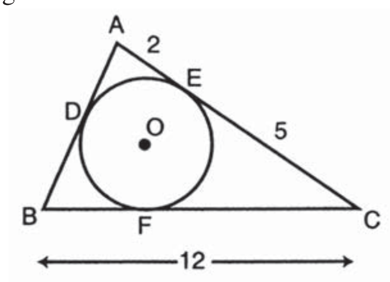
If $\overline{AC}$ and $\overline{AE}$ are tangent to circle O at B and D, then $AB \cong AD$.



Exercise 8

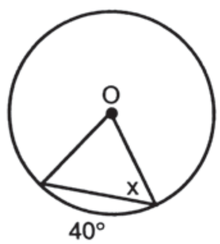
Work out each problem. Circle the letter that appears before your answer.

1. If circle *O* is inscribed in triangle *ABC*, find the length of side *AB*.



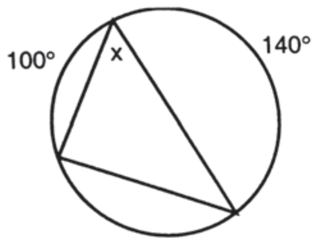
(A) 12
(B) 14
(C) 9
(D) 10
(E) 7

2. Find angle *x*.



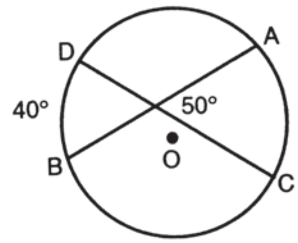
(A) 40°
(B) 20°
(C) 50°
(D) 70°
(E) 80°

3. Find angle *x*.



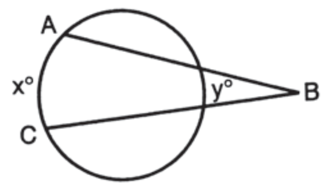
(A) 120°
(B) 50°
(C) 70°
(D) 40°
(E) 60°

4. Find the number of degrees in arc *AC*.



(A) 60
(B) 50
(C) 25
(D) 100
(E) 20

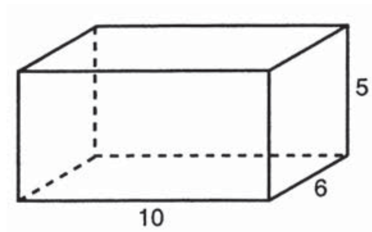
5. The number of degrees in angle *ABC* is



(A) $\frac{1}{2}y$
(B) y
(C) $\frac{1}{2}x$
(D) $\frac{1}{2}(x-y)$
(E) $\frac{1}{2}(x+y)$

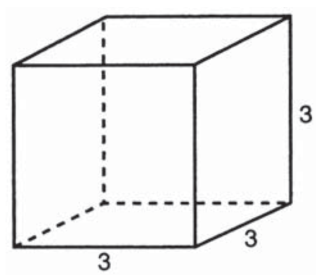
9. VOLUMES

A. The volume of a rectangular solid is equal to the product of its length, width, and height.



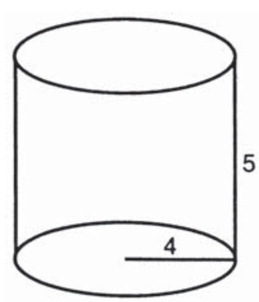
$V = (10)(6)(5) = 300$

B. The volume of a cube is equal to the cube of an edge, since the length, width, and height are all equal.



$V = (3)^3 = 27$

C. The volume of a cylinder is equal to π times the square of the radius of the base times the height.



$V = \pi\ (4)^2\ (5) = 80\pi$

Exercise 9

Work out each problem. Circle the letter that appears before your answer.

1. The surface area of a cube is 96 square feet. How many cubic feet are there in the volume of the cube?
   (A) 16
   (B) 4
   (C) 12
   (D) 64
   (E) 32

2. A cylindrical pail has a radius of 7 inches and a height of 10 inches. Approximately how many gallons will the pail hold if there are 231 cubic inches to a gallon? (Use $\pi = \frac{22}{7}$)
   (A) .9
   (B) 4.2
   (C) 6.7
   (D) 5.1
   (E) 4.8

3. Water is poured into a cylindrical tank at the rate of 9 cubic inches a minute. How many minutes will it take to fill the tank if its radius is 3 inches and its height is 14 inches? (Use $\pi = \frac{22}{7}$)
   (A) $14\frac{2}{3}$
   (B) 44
   (C) 30
   (D) $27\frac{2}{9}$
   (E) 35

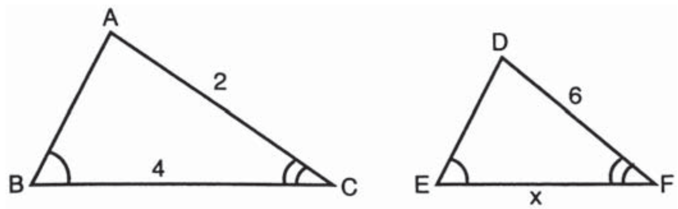
4. A rectangular tank 10 inches by 8 inches by 4 inches is filled with water. If the water is to be transferred to smaller tanks in the form of cubes 4 inches on a side, how many of these tanks are needed?
   (A) 4
   (B) 5
   (C) 6
   (D) 7
   (E) 8

5. The base of a rectangular tank is 6 feet by 5 feet and its height is 16 inches. Find the number of cubic feet of water in the tank when it is $\frac{5}{8}$ full.
   (A) 25
   (B) 40
   (C) 480
   (D) 768
   (E) 300

10. SIMILAR POLYGONS

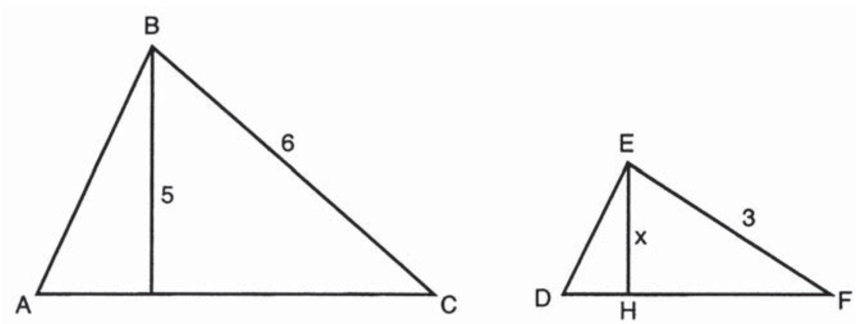
A. Corresponding angles of similar polygons are congruent.

B. Corresponding sides of similar polygons are in proportion.



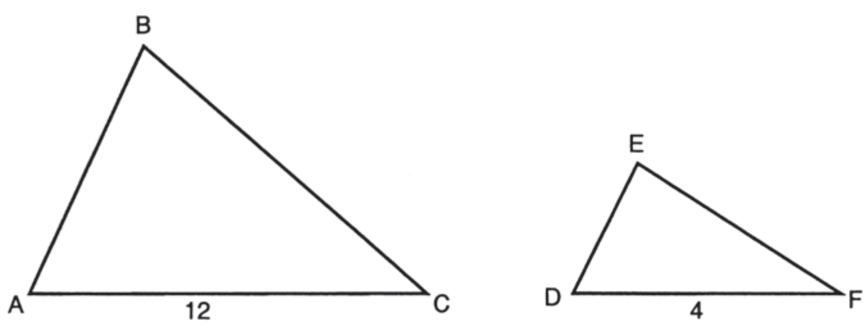
If triangle *ABC* is similar to triangle *DEF* and the sides and angles are given as marked, then *EF* must be equal to 12 as the ratio of corresponding sides is 2 : 6 or 1 : 3.

C. When figures are similar, all ratios between corresponding lines are equal. This includes the ratios of corresponding sides, medians, altitudes, angle bisectors, radii, diameters, perimeters, and circumferences. The ratio is referred to as the linear ratio or ratio of similitude.



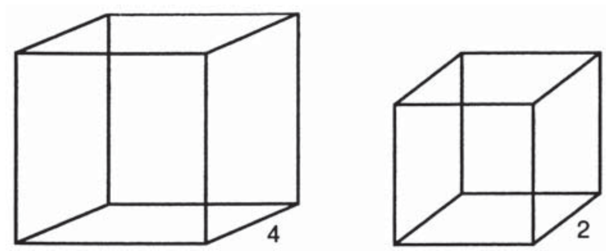
If triangle *ABC* is similar to triangle *DEF* and the segments are given as marked, then *EH* is equal to 2.5 because the linear ratio is 6 : 3 or 2 : 1.

D. When figures are similar, the ratio of their areas is equal to the square of the linear ratio.



If triangle *ABC* is similar to triangle *DEF*, the area of triangle *ABC* will be 9 times as great as the area of triangle *DEF*. The linear ratio is 12 : 4 or 3 : 1. The area ratio will be the square of this or 9 : 1. If the area of triangle *ABC* had been given as 27, the area of triangle *DEF* would be 3.

E. When figures are similar, the ratio of their volumes is equal to the cube of their linear ratio.



The volume of the larger cube is 8 times the volume of the smaller cube. The ratio of sides is 4 : 2 or 2 : 1. The ratio of areas would be 4 : 1. The ratio of volumes would be 8 : 1.

Exercise 10

Work out each problem. Circle the letter that appears before your answer.

1. If the area of a circle of radius x is 5π, find the area of a circle of radius $3x$.
   (A) 10π
   (B) 15π
   (C) 20π
   (D) 30π
   (E) 45π

2. If the length and width of a rectangle are each doubled, the area is increased by
   (A) 50%
   (B) 100%
   (C) 200%
   (D) 300%
   (E) 400%

3. The area of one circle is 9 times as great as the area of another. If the radius of the smaller circle is 3, find the radius of the larger circle.
   (A) 9
   (B) 12
   (C) 18
   (D) 24
   (E) 27

4. If the radius of a circle is doubled, then
   (A) the circumference and area are both doubled
   (B) the circumference is doubled and the area is multiplied by 4
   (C) the circumference is multiplied by 4 and the area is doubled
   (D) the circumference and area are each multiplied by 4
   (E) the circumference stays the same and the area is doubled

5. The volumes of two similar solids are 250 and 128. If a dimension of the larger solid is 25, find the corresponding side of the smaller solid.
   (A) 12.8
   (B) 15
   (C) 20
   (D) 40
   (E) cannot be determined