1. SIMPLIFYING FRACTIONS

In simplifying fractions, we must divide the numerator and denominator by the same factor. We can multiply or divide both the numerator and denominator of a fraction by the same number without changing the value of the fraction. However, if we were to add or subtract the same number in the numerator and denominator, the value of the fraction would not remain the same. When we simplify $\frac{9}{12}$ to $\frac{3}{4}$, we are really saying that $\frac{9}{12}=\frac{3\cdot 3}{3\cdot 4}$ and then dividing the numerator and denominator by 3. We may not say that $\frac{9}{12}=\frac{5+4}{5+7}$ and then say that $\frac{9}{12}=\frac{4}{7}$. This is a serious error in algebra as well. $\frac{9t}{12t}=\frac{3}{4}$ because we divide numerator and denominator by $3t$. However, $\frac{9+t}{12+t}$ cannot be simplified, as there is no factor that divides into the *entire* numerator as well as the *entire* denominator. *Never cancel terms!* That is, never cancel parts of numerators or denominators containing + or – signs unless they are enclosed in parentheses as parts of factors. This is one of the most frequent student errors. Be very careful to avoid it.

Example:

Simplify $\frac{4b^2+8b}{3b^3+6b^2}$

Solution:

Factoring the numerator and denominator by removing the largest common factor in both cases, we have $\frac{4b(b+2)}{3b^2(b+2)}$

The factors common to both numerator and denominator are b and $(b + 2)$. Dividing these out, we have $\frac{4}{3b}$.

Example:

Simplify $\frac{x^2+6x+8}{x^2+x-12}$ to simplest form.

Solution:

There are no common factors here, but both numerator and denominator may be factored as trinomials. $\frac{(x+4)(x+2)}{(x+4)(x-3)}$ gives $\frac{(x+2)}{(x-3)}$ as a final answer. Remember not to cancel the x's as they are *terms* and not *factors*.

Example:

Simplify $\frac{10-2x}{x^2-4x-5}$ to simplest form.

Solution:

The numerator contains a common factor, while the denominator must be factored as a trinomial.

$$2\left(\frac{5-x}{(x-5)(x+1)}\right)$$

When numbers are reversed around a minus sign, they may be turned around by factoring out a (–1).5 – x = (– 1)(x – 5). Doing this will enable us to simplify the fraction to $\frac{-2}{x+1}$. Remember that if the terms had been reversed around a plus sign, the factors could have been divided without factoring further, as $a + b = b + a$, by the cummutative law of addition. Subtraction, however, is not commutative, necessitating the factoring of –1.

Exercise 1

Work out each problem. Circle the letter that appears before your answer.

1. Simplify to simplest form: $\frac{3x^3 - 3x^2y}{9x^2 - 9xy}$

   (A) $\frac{x}{6}$

   (B) $\frac{x}{3}$

   (C) $\frac{2x}{3}$

   (D) 1

   (E) $\frac{x-y}{3}$

2. Simplify to simplest form: $\frac{2x-8}{12-3x}$

   (A) $-\frac{2}{3}$

   (B) $\frac{2}{3}$

   (C) $-\frac{4}{3}$

   (D) $\frac{4}{3}$

   (E) $-\frac{3}{2}$

3. Find the value of $\frac{3x-y}{y-3x}$ when $x = \frac{2}{7}$ and $y = \frac{3}{10}$.

   (A) $\frac{24}{70}$

   (B) $\frac{11}{70}$

   (C) 0

   (D) 1

   (E) –1

4. Simplify to simplest form: $\frac{b^2 + b - 12}{b^2 + 2b - 15}$

   (A) $\frac{4}{5}$

   (B) $-\frac{4}{3}$

   (C) $\frac{b+4}{b+5}$

   (D) $\frac{b-4}{b-5}$

   (E) $-\frac{b+4}{b+5}$

5. Simplify to simplest form: $\frac{2x+4y}{6x+12y}$

   (A) $\frac{2}{3}$

   (B) $-\frac{2}{3}$

   (C) $-\frac{1}{3}$

   (D) $\frac{1}{3}$

   (E) 3

2. ADDITION OR SUBTRACTION OF FRACTIONS

In adding or subtracting fractions, it is necessary to have the fractions expressed in terms of the same common denominator. When adding or subtracting two fractions, use the same shortcuts used in arithmetic. Remember that $\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$, and that $\frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd}$. All sums or differences should be simplified to simplest form.

Example:

Add $\frac{3}{a}+\frac{2}{b}$

Solution:

Add the two cross products and put the sum over the denominator product: $\frac{3b+2a}{ab}$

Example:

Add $\frac{2a}{3}+\frac{4a}{5}$

Solution:

$$\frac{10a+12a}{15}=\frac{22a}{15}$$

Example:

Add $\frac{5a}{a+b}+\frac{5b}{a+b}$

Solution:

Since both fractions have the same denominator, we must simply add the numerators and put the sum over the same denominator.

$$\frac{5a+5b}{a+b}=\frac{5(a+b)}{a+b}=5$$

Example:

Subtract $\frac{4r-s}{6}-\frac{2r-7s}{6}$

Solution:

Since both fractions have the same denominator, we subtract the numerators and place the difference over the same denominator. Be very careful of the minus sign between the fractions as it will change the sign of each term in the second numerator.

$$\frac{4r-s-(2r-7s)}{6}=\frac{4r-s-2r+7s}{6}=\frac{2r+6s}{6}=\frac{2(r+3s)}{6}=\frac{r+3s}{3}$$

Exercise 2

Work out each problem. Circle the letter that appears before your answer.

1. Subtract $\frac{6x+5y}{2x} - \frac{4x+y}{2x}$
   (A) $1 + 4y$
   (B) $4y$
   (C) $1 + 2y$
   (D) $\frac{x+2y}{x}$
   (E) $\frac{x+3y}{x}$

2. Add $\frac{3c}{c+d} + \frac{3d}{c+d}$
   (A) $\frac{6cd}{c+d}$
   (B) $\frac{3cd}{c+d}$
   (C) $\frac{3}{2}$
   (D) 3
   (E) $\frac{9cd}{c+d}$

3. Add $\frac{a}{5} + \frac{3a}{10}$
   (A) $\frac{4a}{15}$
   (B) $\frac{a}{2}$
   (C) $\frac{3a^2}{50}$
   (D) $\frac{2a}{25}$
   (E) $\frac{3a^2}{15}$

4. Add $\frac{x+4}{6} + \frac{1}{2}$
   (A) $\frac{x+7}{6}$
   (B) $\frac{x+5}{8}$
   (C) $\frac{x+4}{12}$
   (D) $\frac{x+5}{12}$
   (E) $\frac{x+5}{6}$

5. Subtract $\frac{3b}{4} - \frac{7b}{10}$
   (A) $-\frac{2b}{3}$
   (B) $\frac{b}{5}$
   (C) $\frac{b}{20}$
   (D) b
   (E) $\frac{2b}{3}$

3. MULTIPLICATION OR DIVISION OF FRACTIONS

In multiplying or dividing fractions, we must first factor all numerators and denominators and may then divide all factors common to any numerator and any denominator. Remember always to invert the fraction following the division sign. Where exponents are involved, they are added in multiplication and subtracted in division.

Example:

Find the product of $\frac{x^3}{y^2}$ and $\frac{y^3}{x^2}$.

Solution:

Factors common to both numerator and denominator are x^2 in the first numerator and second denominator and y^2 in the first denominator and second numerator. Dividing by these common factors, we are left with $\frac{x}{1} \cdot \frac{y}{1}$. Finally, we multiply the resulting fractions, giving an answer of xy.

Example:

Divide $\frac{15a^2b}{2}$ by $5a^3$.

Solution:

We invert the divisor and multiply.

$\frac{15a^2b}{2} \cdot \frac{1}{5a^3}$

We can divide the first numerator and second denominator by $5a^2$, giving $\frac{3b}{2} \cdot \frac{1}{a}$ or $\frac{3b}{2a}$.

Exercise 3

Work out each problem. Circle the letter that appears before your answer.

1. Find the product of $\frac{x^2}{y^3}$ and $\frac{y^4}{x^5}$
   (A) $\frac{y^2}{x^3}$
   (B) $\frac{y}{x^3}$
   (C) $\frac{x^3}{y}$
   (D) $\frac{x^8}{y^7}$
   (E) $\frac{x}{y}$

2. Multiply c by $\frac{b}{c}$
   (A) $\frac{b}{c^2}$
   (B) $\frac{c^2}{b}$
   (C) b
   (D) c
   (E) bc^2

3. Divide $\frac{ax}{by}$ by $\frac{x}{y}$
   (A) $\frac{ax^2}{by^2}$
   (B) $\frac{b}{a}$
   (C) $\frac{a}{b}$
   (D) $\frac{by^2}{ax^2}$
   (E) $\frac{ay}{bx}$

4. Divide $4abc$ by $\frac{2a^2b}{3d^2}$
   (A) $\frac{8a^3b^2c}{3d^2}$
   (B) $\frac{a}{6cd^2}$
   (C) $\frac{2ac}{bd^2}$
   (D) $\frac{6cd^2}{a}$
   (E) $\frac{5cd^2}{a}$

5. Divide $\frac{3a^2c^4}{4b^2}$ by $6ac^2$
   (A) $\frac{ac^2}{8b^2}$
   (B) $\frac{ac^2}{4b^2}$
   (C) $\frac{4b^2}{ac^2}$
   (D) $\frac{8b^2}{ac^2}$
   (E) $\frac{ac^2}{6b^2}$

4. COMPLEX ALGEBRAIC FRACTIONS

Complex algebraic fractions are simplified by the same methods reviewed earlier for arithmetic fractions. To eliminate the fractions within the fraction, multiply *each term* of the entire complex fraction by the lowest quantity that will eliminate them all.

Example:

Simplify $\dfrac{\frac{3}{x}+\frac{2}{y}}{6}$

Solution:

We must multiply *each term* by xy, giving $\dfrac{3y+2x}{6xy}$.

No more simplification is possible beyond this. Remember *never* to cancel terms or parts of terms. We may only simplify by dividing factors.

Exercise 4

Work out each problem. Circle the letter that appears before your answer.

1. Simplify $\dfrac{\frac{1}{5}-\frac{3}{2}}{\frac{3}{4}}$

   (A) $\frac{15}{26}$

   (B) $-\frac{15}{26}$

   (C) 2

   (D) $\frac{26}{15}$

   (E) $-\frac{26}{15}$

2. Simplify $\dfrac{\frac{a}{x^2}}{\frac{a^2}{x}}$

   (A) $\frac{x}{a}$

   (B) $\frac{1}{a^2x}$

   (C) $\frac{1}{ax}$

   (D) ax

   (E) $\frac{a}{x}$

3. Simplify $\dfrac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{x}+\frac{1}{y}}$

   (A) $\frac{x-y}{x+y}$

   (B) $\frac{x+y}{x-y}$

   (C) $\frac{y-x}{x+y}$

   (D) -1

   (E) $-xy$

4. Simplify $1+\dfrac{\frac{1}{x}}{\frac{1}{y}}$

   (A) $\frac{x+y}{x}$

   (B) $2y$

   (C) $x + 1$

   (D) $\frac{y+1}{x}$

   (E) $\frac{x+1}{y}$

5. Simplify $2+\dfrac{\frac{1}{t}}{\frac{2}{t^2}}$

   (A) $t^2 + t$

   (B) t^3

   (C) $\frac{2t+1}{2}$

   (D) $t + 1$

   (E) $\frac{4+t}{2}$

5. USING FACTORING TO FIND MISSING VALUES

Certain types of problems may involve the ability to factor in order to evaluate a given expression. In particular, you should be able to factor the difference of two perfect squares. If an expression consists of two terms that are separated by a minus sign, the expression can always be factored into two binomials, with one containing the sum of the square roots and the other their difference. This can be stated by the identity $x^2 - y^2 = (x + y)(x - y)$.

Example:

If $m^2 - n^2 = 48$ and $m + n = 12$, find $m - n$.

Solution:

Since $m^2 - n^2$ is equal to $(m + n)(m - n)$, these two factors must multiply to 48. If one of them is 12, the other must be 4.

Example:

If $(a + b)^2 = 48$ and $ab = 6$, find $a^2 + b^2$.

Solution:

$(a + b)^2$ is equal to $a^2 + 2ab + b^2$. Substituting 6 for ab, we have $a^2 + 2(6) + b^2 = 48$ and $a^2 + b^2 = 36$.

Exercise 5

Work out each problem. Circle the letter that appears before your answer.

1. If $a+b=\frac{1}{3}$ and $a-b=\frac{1}{4}$, find $a^2 - b^2$.
   (A) $\frac{1}{12}$
   (B) $\frac{1}{7}$
   (C) $\frac{2}{7}$
   (D) $\frac{1}{6}$
   (E) none of these

2. If $(a - b)^2 = 40$ and $ab = 8$, find $a^2 + b^2$.
   (A) 5
   (B) 24
   (C) 48
   (D) 56
   (E) 32

3. If $a + b = 8$ and $a^2 - b^2 = 24$, then $a - b =$
   (A) 16
   (B) 4
   (C) 3
   (D) 32
   (E) 6

4. The trinomial $x^2 + 4x - 45$ is exactly divisible by
   (A) $x + 9$
   (B) $x - 9$
   (C) $x + 5$
   (D) $x + 15$
   (E) $x - 3$

5. If $\frac{1}{c}-\frac{1}{d}=5$ and $\frac{1}{c}+\frac{1}{d}=3$, then $\frac{1}{c^2}-\frac{1}{d^2}=$
   (A) 16
   (B) 34
   (C) 2
   (D) 15
   (E) cannot be determined