1. ADDITION AND SUBTRACTION OF RADICALS

The conditions under which radicals can be added or subtracted are much the same as the conditions for letters in an algebraic expression. The radicals act as a label, or unit, and must therefore be exactly the same. In adding or subtracting, we add or subtract the coefficients, or rational parts, and carry the radical along as a label, which does not change.

Example:

$\sqrt{2}+\sqrt{3}$ cannot be added

$\sqrt{2}+\sqrt[3]{2}$ cannot be added

$4\sqrt{2}+5\sqrt{2}=9\sqrt{2}$

Often, when radicals to be added or subtracted are not the same, simplification of one or more radicals will make them the same. To simplify a radical, we remove any perfect square factors from underneath the radical sign.

Example:

$\sqrt{12}=\sqrt{4}\cdot\sqrt{3}=2\sqrt{3}$

$\sqrt{27}=\sqrt{9}\cdot\sqrt{3}=3\sqrt{3}$

If we wish to add $\sqrt{12}+\sqrt{27}$, we must first simplify each one. Adding the simplified radicals gives a sum of $5\sqrt{3}$.

Example:

$\sqrt{125}+\sqrt{20}-\sqrt{500}$

Solution:

$\sqrt{25}\cdot\sqrt{5}+\sqrt{4}\cdot\sqrt{5}-\sqrt{100}\cdot\sqrt{5}$

$=5\sqrt{5}+2\sqrt{5}-10\sqrt{5}$

$=-3\sqrt{5}$

Exercise 1

Work out each problem. Circle the letter that appears before your answer.

1. Combine $4\sqrt{27} - 2\sqrt{48} + \sqrt{147}$
   (A) $27\sqrt{3}$
   (B) $-3\sqrt{3}$
   (C) $9\sqrt{3}$
   (D) $10\sqrt{3}$
   (E) $11\sqrt{3}$

2. Combine $\sqrt{80} + \sqrt{45} - \sqrt{20}$
   (A) $9\sqrt{5}$
   (B) $5\sqrt{5}$
   (C) $-\sqrt{5}$
   (D) $3\sqrt{5}$
   (E) $-2\sqrt{5}$

3. Combine $6\sqrt{5} + 3\sqrt{2} - 4\sqrt{5} + \sqrt{2}$
   (A) 8
   (B) $2\sqrt{5} + 3\sqrt{2}$
   (C) $2\sqrt{5} + 4\sqrt{2}$
   (D) $5\sqrt{7}$
   (E) 5

4. Combine $\frac{1}{2}\cdot\sqrt{180} + \frac{1}{3}\cdot\sqrt{45} - \frac{2}{5}\cdot\sqrt{20}$
   (A) $3\sqrt{10} + \sqrt{15} + 2\sqrt{2}$
   (B) $\frac{16}{5}\sqrt{5}$
   (C) $\sqrt{97}$
   (D) $\frac{24}{5}\sqrt{5}$
   (E) none of these

5. Combine $5\sqrt{mn} - 3\sqrt{mn} - 2\sqrt{mn}$
   (A) 0
   (B) 1
   (C) $\sqrt{mn}$
   (D) mn
   (E) $-\sqrt{mn}$

2. MULTIPLICATION AND DIVISION OF RADICALS

In multiplication and division, we again treat the radicals as we would treat letters in an algebraic expression. They are factors and must be treated as such.

Example:

$$\sqrt{2} \cdot \sqrt{3} = \sqrt{6}$$

Example:

$$4\sqrt{2} \cdot 5\sqrt{3} = 20 \cdot \sqrt{6}$$

Example:

$$(3\sqrt{2})^2 = 3\sqrt{2} \cdot 3\sqrt{2} = 9 \cdot 2 = 18$$

Example:

$$\frac{\sqrt{8}}{\sqrt{2}} = \sqrt{4} = 2$$

Example:

$$\frac{10\sqrt{20}}{2\sqrt{4}} = 5\sqrt{5}$$

Example:

$$\sqrt{2}(\sqrt{8} + \sqrt{18}) = \sqrt{16} + \sqrt{36} = 4 + 6 = 10$$

Exercise 2

Work out each problem. Circle the letter that appears before your answer.

1. Multiply and simplify: $2\sqrt{18} \cdot 6\sqrt{2}$
   (A) 72
   (B) 48
   (C) $12\sqrt{6}$
   (D) $8\sqrt{6}$
   (E) 36

2. Find $\left(3\sqrt{3}\right)^3$
   (A) $27\sqrt{3}$
   (B) $81\sqrt{3}$
   (C) 81
   (D) $9\sqrt{3}$
   (E) 243

3. Multiply and simplify: $\frac{1}{2}\sqrt{2}(\sqrt{6} + \frac{1}{2}\sqrt{2})$
   (A) $\sqrt{3} + \frac{1}{2}$
   (B) $\frac{1}{2} \cdot \sqrt{3}$
   (C) $\sqrt{6} + 1$
   (D) $\sqrt{6} + \frac{1}{.2}$
   (E) $\sqrt{6} + 2$

4. Divide and simplify: $\frac{\sqrt{32b^3}}{\sqrt{8b}}$
   (A) $2\sqrt{b}$
   (B) $\sqrt{2b}$
   (C) $2b$
   (D) $\sqrt{2b^2}$
   (E) $b\sqrt{2b}$

5. Divide and simplify: $\frac{15\sqrt{96}}{5\sqrt{2}}$
   (A) $7\sqrt{3}$
   (B) $7\sqrt{12}$
   (C) $11\sqrt{3}$
   (D) $12\sqrt{3}$
   (E) $40\sqrt{3}$

3. SIMPLIFYING RADICALS CONTAINING A SUM OR DIFFERENCE

In simplifying radicals that contain several terms under the radical sign, we must combine terms before taking the square root.

Example:

$\sqrt{16+9} = \sqrt{25} = 5$

It is not true that $\sqrt{16+9} = \sqrt{16} + \sqrt{9}$, which would be 4 + 3, or 7.

Example:

$$\sqrt{\frac{x^2}{16} - \frac{x^2}{25}} = \sqrt{\frac{25x^2 - 16x^2}{400}} = \sqrt{\frac{9x^2}{400}} = \frac{3x}{20}$$

Exercise 3

Work out each problem. Circle the letter that appears before your answer.

1. Simplify $\sqrt{\frac{x^2}{9} + \frac{x^2}{16}}$
   (A) $\frac{25x^2}{144}$
   (B) $\frac{5x}{12}$
   (C) $\frac{5x^2}{12}$
   (D) $\frac{x}{7}$
   (E) $\frac{7x}{12}$

2. Simplify $\sqrt{36y^2 + 64x^2}$
   (A) $6y + 8x$
   (B) $10xy$
   (C) $6y^2 + 8x^2$
   (D) $10x^2y^2$
   (E) cannot be done

3. Simplify $\sqrt{\frac{x^2}{64} - \frac{x^2}{100}}$
   (A) $\frac{x}{40}$
   (B) $-\frac{x}{2}$
   (C) $\frac{x}{2}$
   (D) $\frac{3x}{40}$
   (E) $\frac{3x}{80}$

4. Simplify $\sqrt{\frac{y^2}{2} - \frac{y^2}{18}}$
   (A) $\frac{2y}{3}$
   (B) $\frac{y}{5}$
   (C) $\frac{10y}{3}$
   (D) $\frac{y\sqrt{3}}{6}$
   (E) cannot be done

5. $\sqrt{a^2 + b^2}$ is equal to
   (A) $a + b$
   (B) $a - b$
   (C) $\sqrt{a^2} + \sqrt{b^2}$
   (D) $(a + b)(a - b)$
   (E) none of these

4. FINDING THE SQUARE ROOT OF A NUMBER

In finding the square root of a number, the first step is to pair off the digits in the square root sign in each direction from the decimal point. If there is an odd number of digits *before* the decimal point, insert a zero at the *beginning* of the number in order to pair digits. If there is an odd number of digits *after* the decimal point, add a zero at the *end*. It should be clearly understood that these zeros are place holders only and in no way change the value of the number. Every *pair* of numbers in the radical sign gives one digit of the square root.

Example:

Find the number of digits in the square root of 328,329.

Solution:

Pair the numbers beginning at the decimal point.

$\sqrt{32\ \ 83\ \ 29}$.

Each pair will give one digit in the square root. Therefore the square root of 328,329 has three digits.

If we were asked to find the square root of 328,329, we would look among the multiple-choice answers for a three-digit number. If there were more than one, we would have to use additional criteria for selection. Since our number ends in 9, its square root must end in a digit that, when multiplied by itself, ends in 9. Going through the digits from 0 to 9, this could be 3 ($3 \cdot 3 = 9$) or 7 ($7 \cdot 7 = 49$). Only one of these would appear among the choices, as this examination will not call for extensive computation, but rather for sound mathematical reasoning.

Example:

The square root of 4624 is exactly

(A) 64
(B) 65
(C) 66
(D) 67
(E) 68

Solution:

Since all choices contain two digits, we must reason using the last digit. It must be a number that, when multiplied by itself, will end in 4. Among the choices, the only possibility is 68 as 64^2 will end in 6, 65^2 will end in 5, 66^2 in 6, and 67^2 in 9.

Exercise 4

Work out each problem. Circle the letter that appears before your answer.

1. The square root of 17,689 is exactly
   (A) 131
   (B) 132
   (C) 133
   (D) 134
   (E) 136

2. The number of digits in the square root of 64,048,009 is
   (A) 4
   (B) 5
   (C) 6
   (D) 7
   (E) 8

3. The square root of 222.01 is exactly
   (A) 14.3
   (B) 14.4
   (C) 14.6
   (D) 14.8
   (E) 14.9

4. The square root of 25.6036 is exactly
   (A) 5.6
   (B) 5.06
   (C) 5.006
   (D) 5.0006
   (E) 5.00006

5. Which of the following square roots can be found exactly?
   (A) $\sqrt{.4}$
   (B) $\sqrt{.9}$
   (C) $\sqrt{.09}$
   (D) $\sqrt{.02}$
   (E) $\sqrt{.025}$